Complex Networks Approach

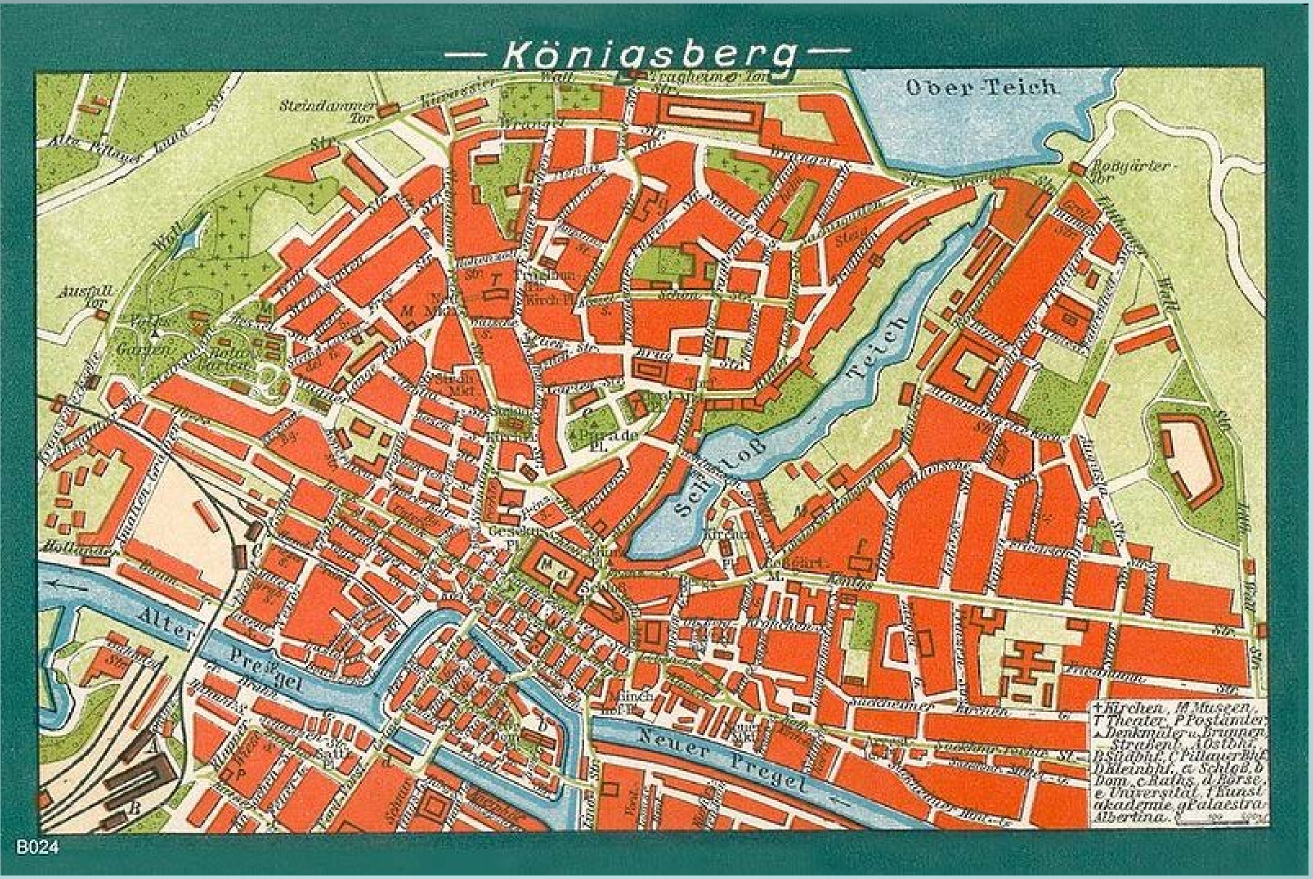
Jürgen Kurths

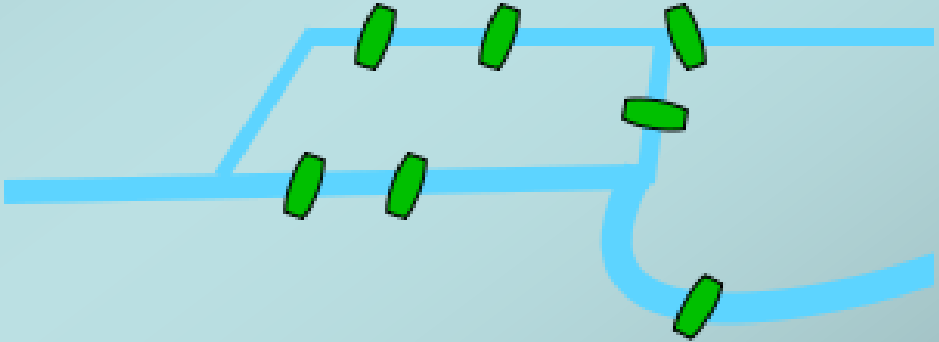
Potsdam Institute for Climate Impact Research
&
Institut of Physics, Humboldt-Universität zu Berlin
&
King‘s College, University of Aberdeen

juergen.kurths@pik-potsdam.de
http://www.pik-potsdam.de/members/kurths/

Networks with Complex Topology

Königsberger Brückenproblem (L. Euler, 1736)

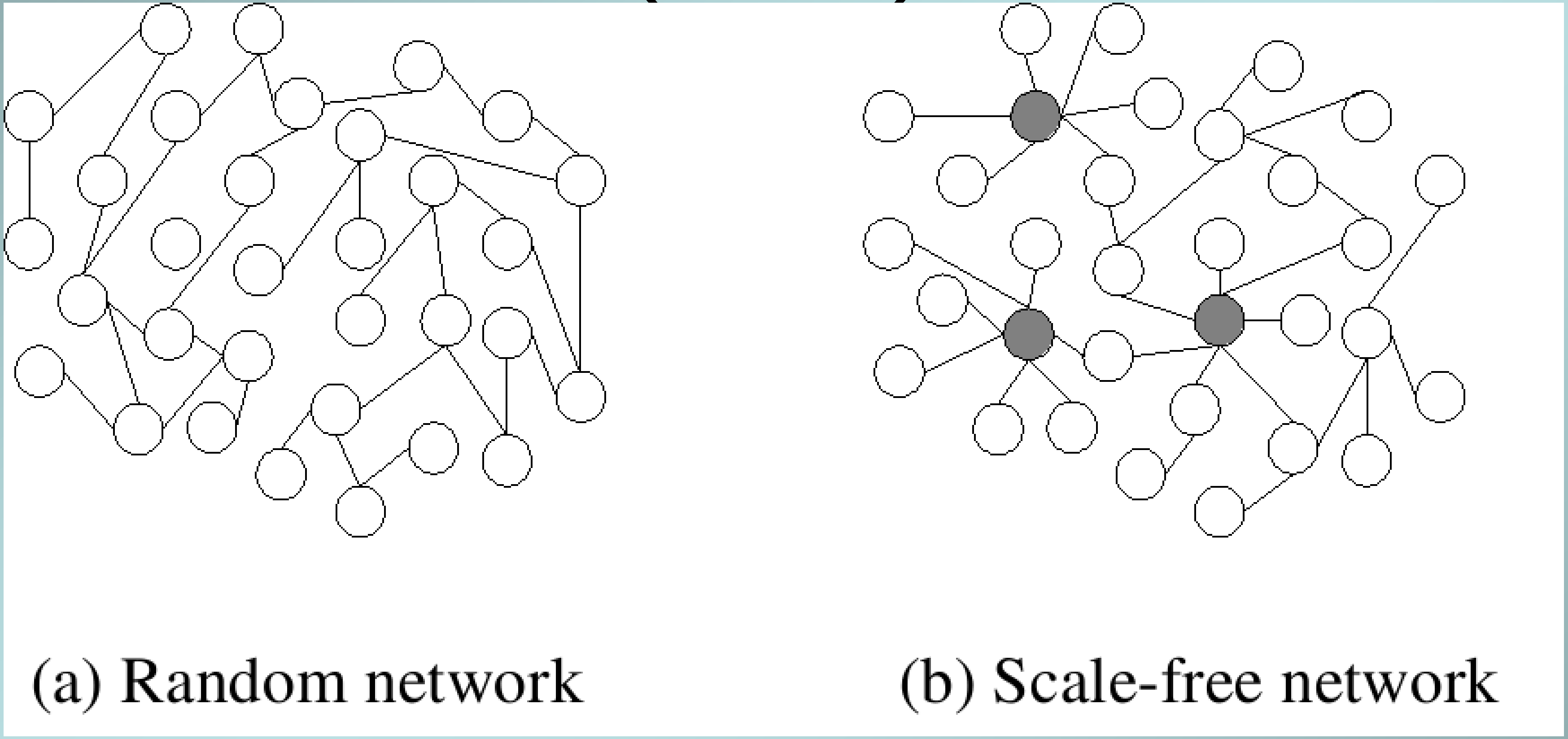




1707 (Basel) – 1783 (St. Petersburg)

1741-66 Berlin

Erdös-Renyi Random Networks (1959)

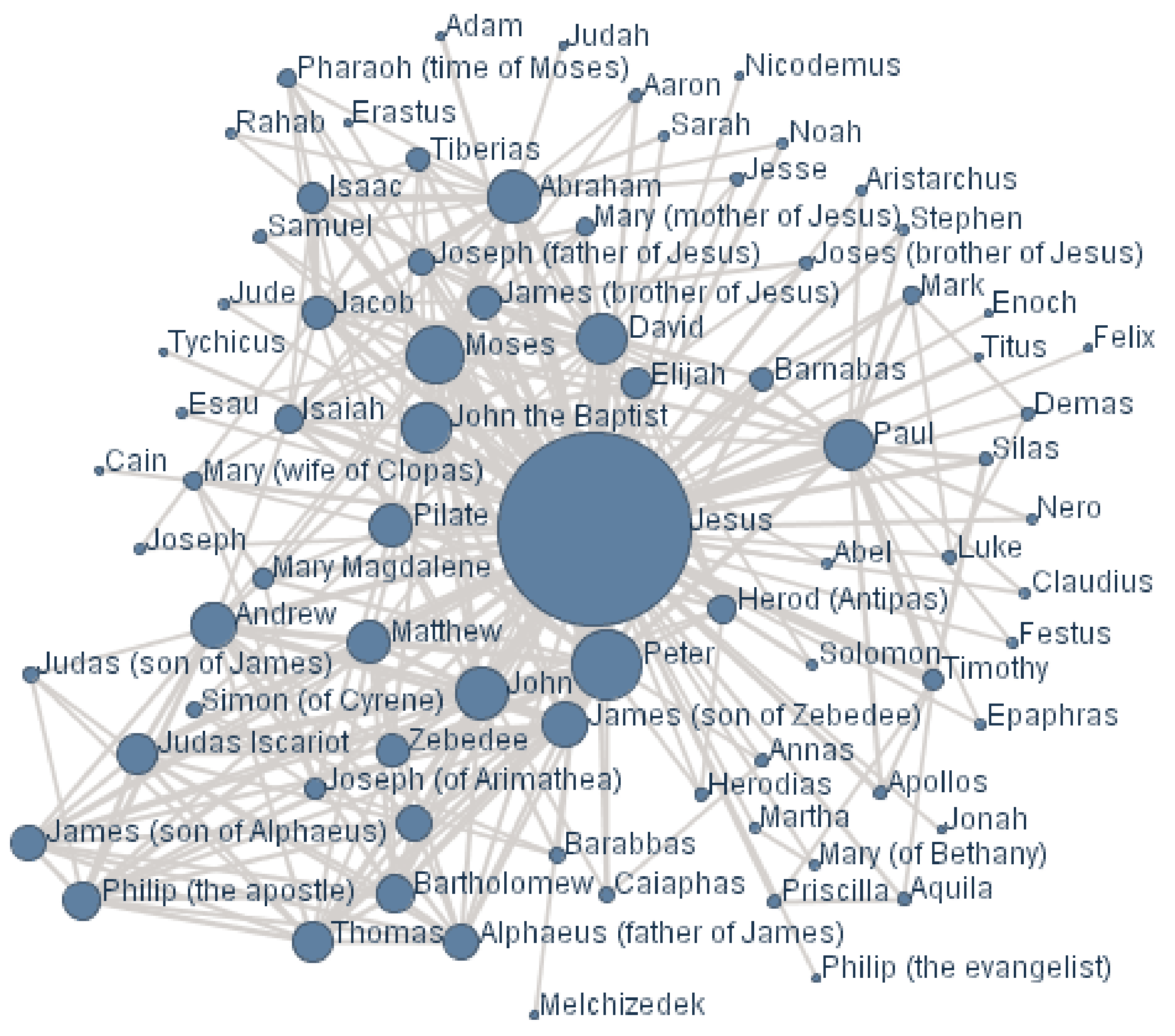


(a) Random network

(b) Scale-free network

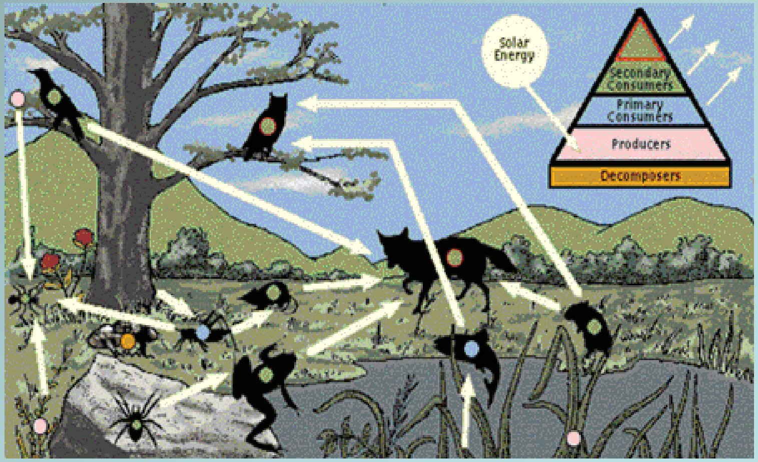
Much mathematical theory about

Social Networks

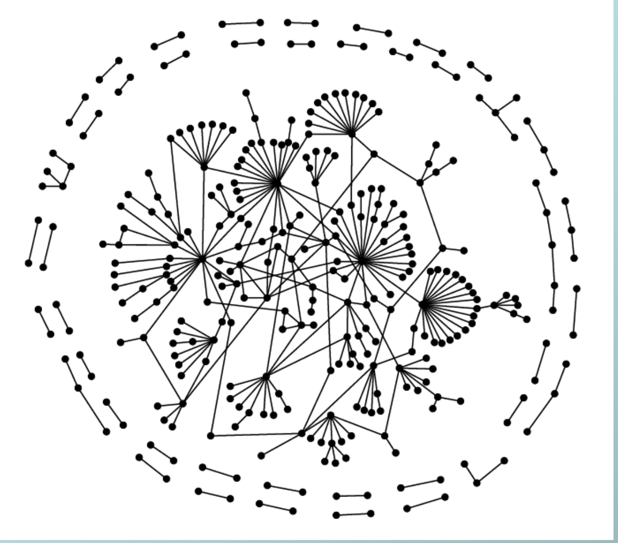


Biological Networks

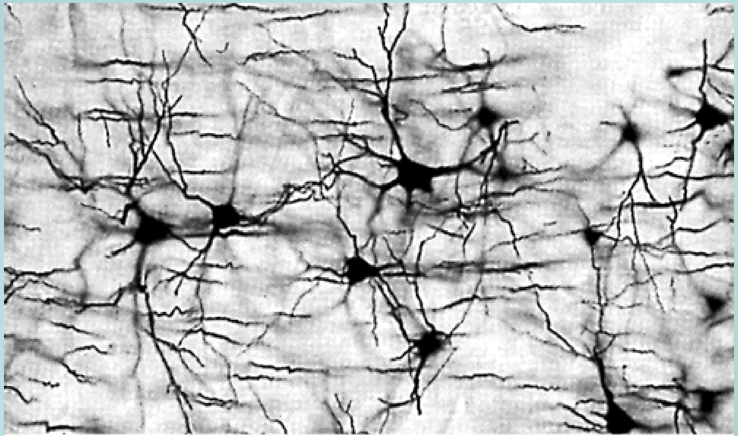
Ecological



Protein interaction



Neural Networks

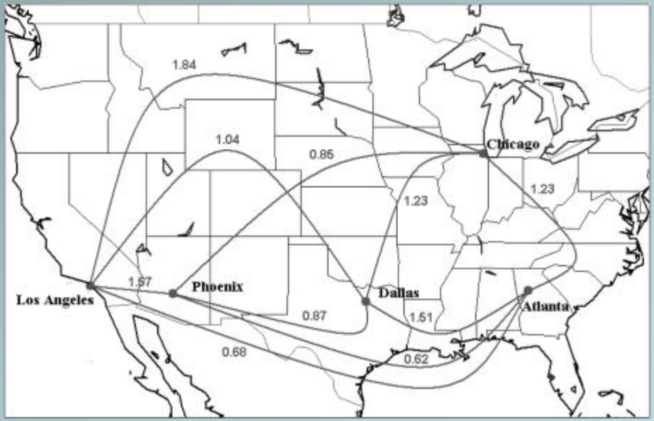


Genetic Networks

Metabolic Networks

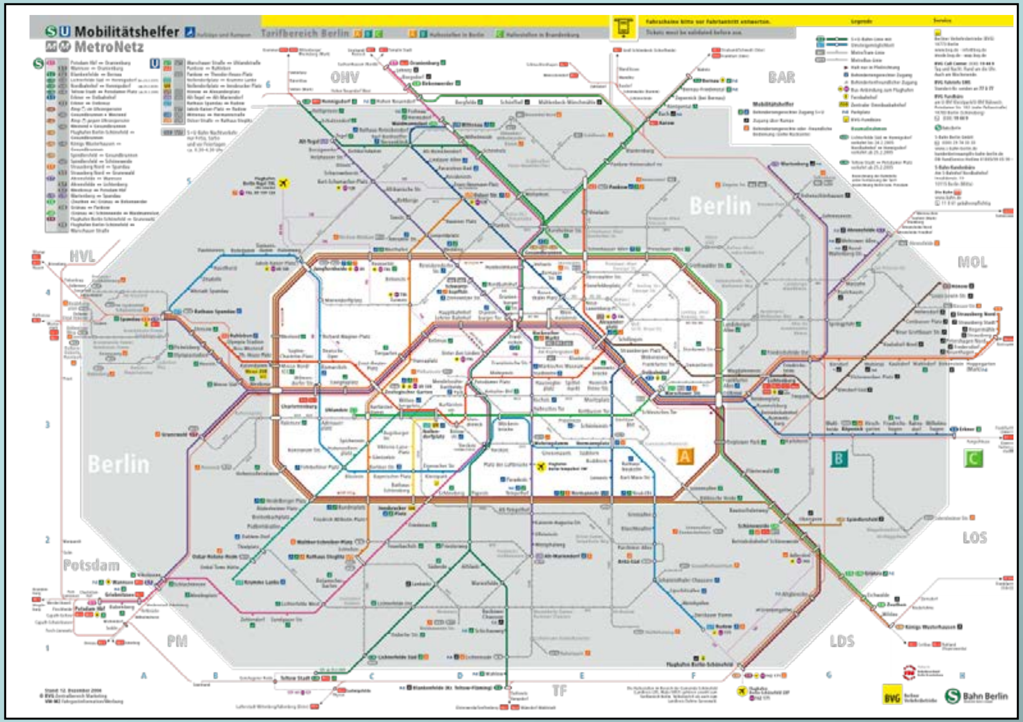
Transportation Networks

Airport Networks

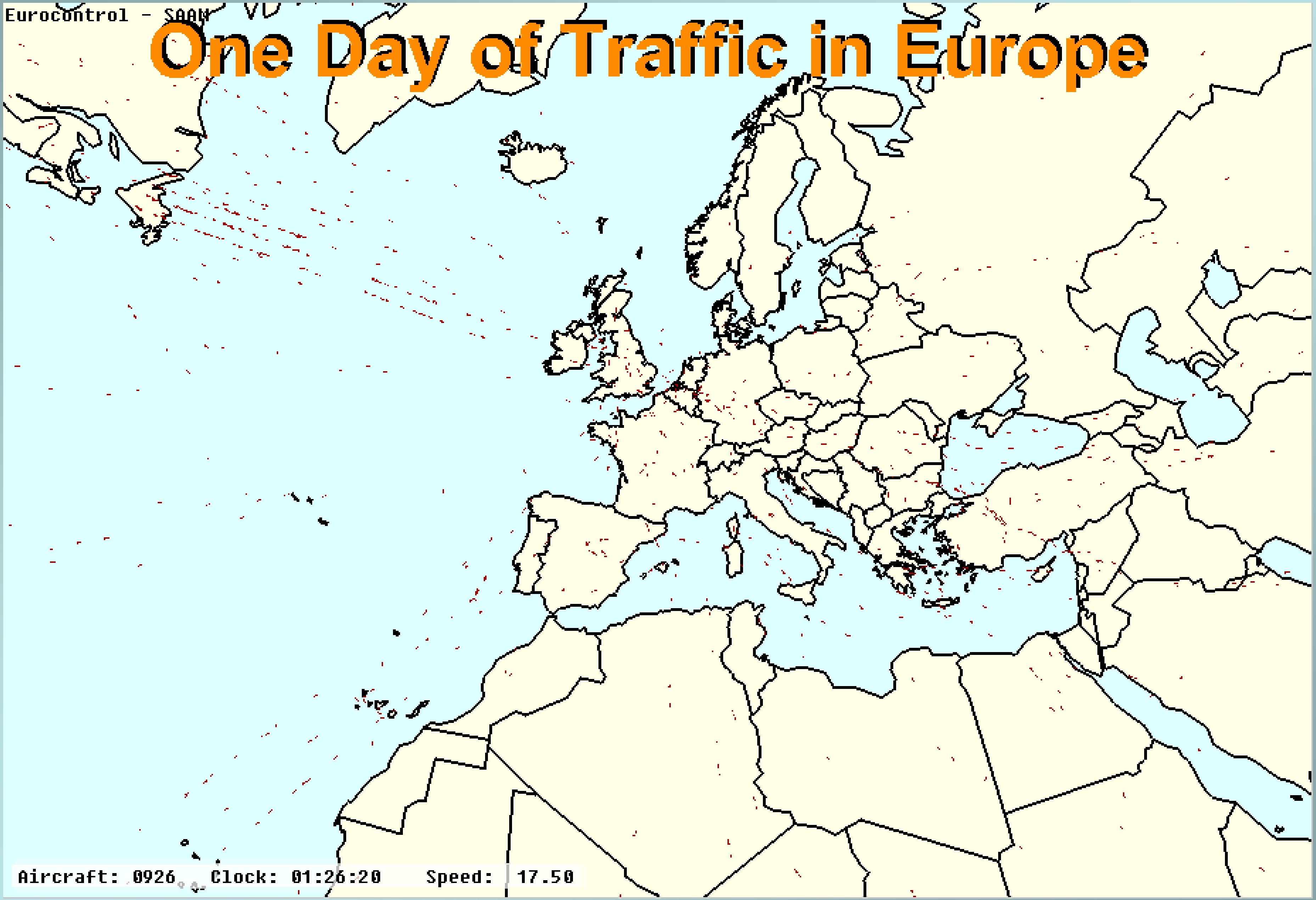


Road Maps

Local



Evolving Networks



Network of Networks

Interconnected Networks

Interdependent Networks

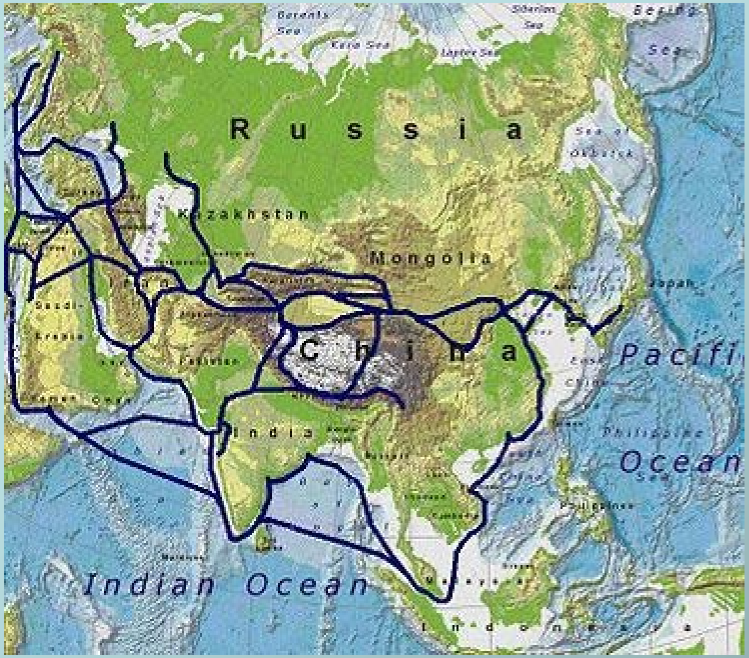
Interacting Networks

Multilayer Networks

Transportation Networks

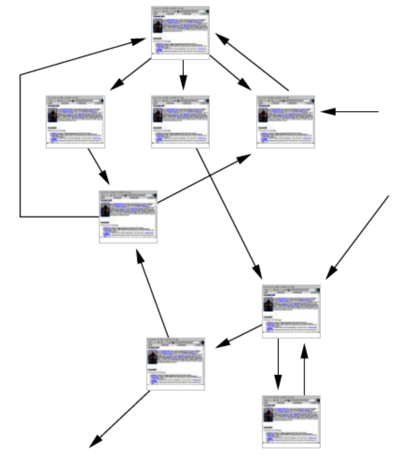
- Romans built > 850.000 km roads (Network)
- „Silk Street“

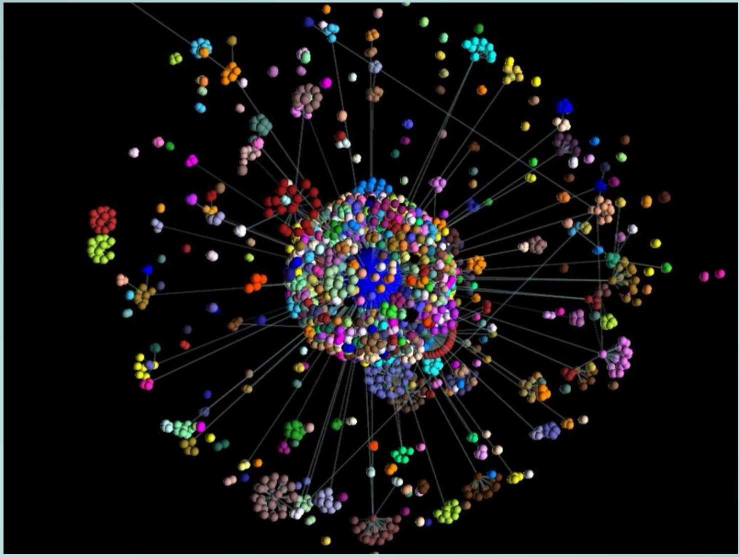
  (Network)



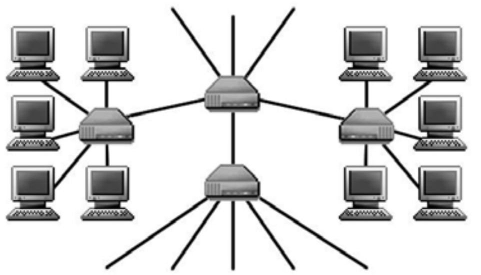
Technological Networks

World-Wide Web

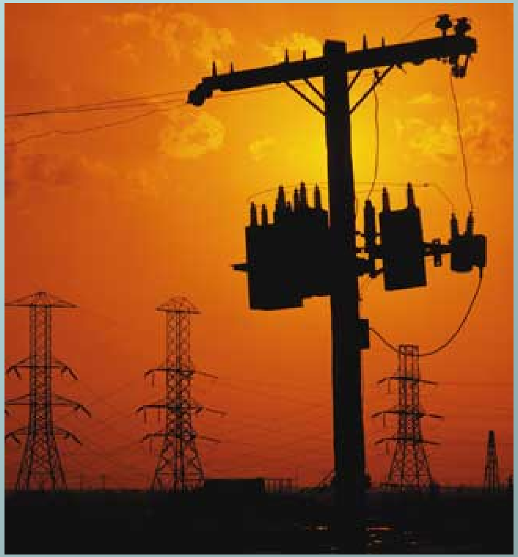




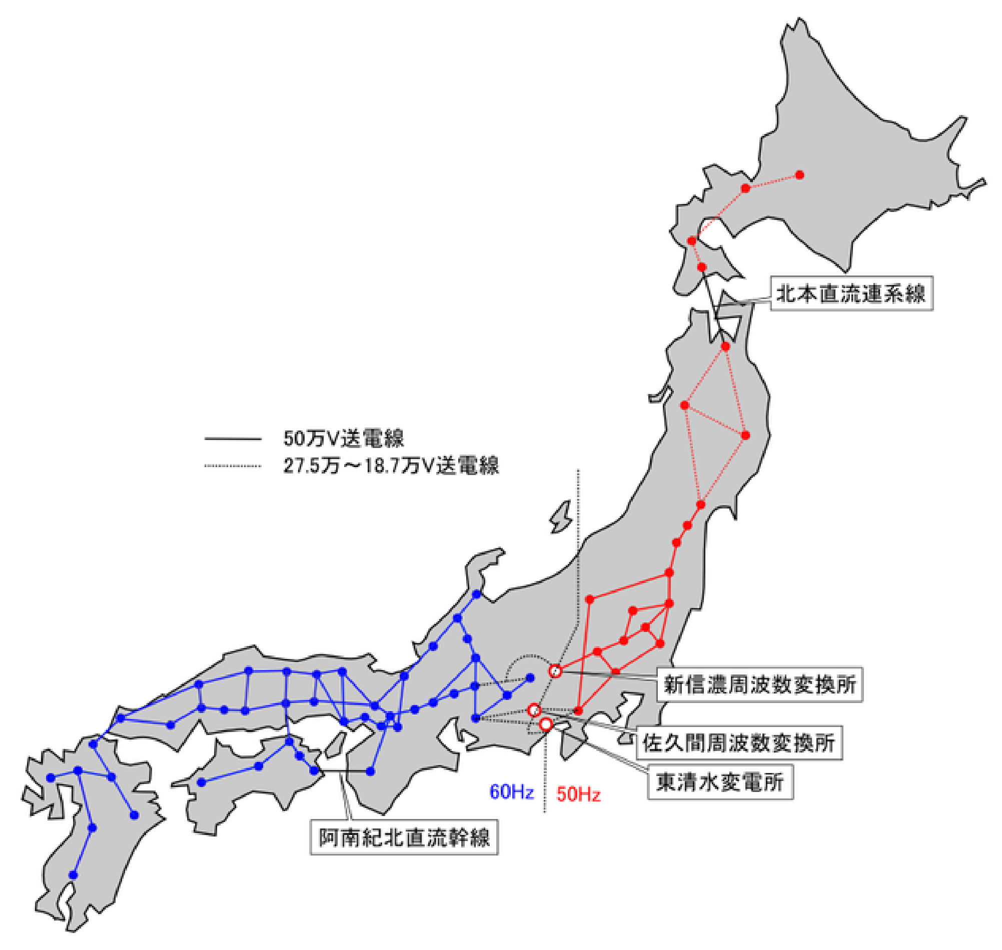
Internet



Power Grid



Power grid in Japan



How to control such networks?

Pinning Control (which nodes?)

Highly Non-trivial Task

Papenburg: Monster Black-Out 06-11-2006

- Meyer Werft in Papenburg
- Newly built ship Norwegian Pearl

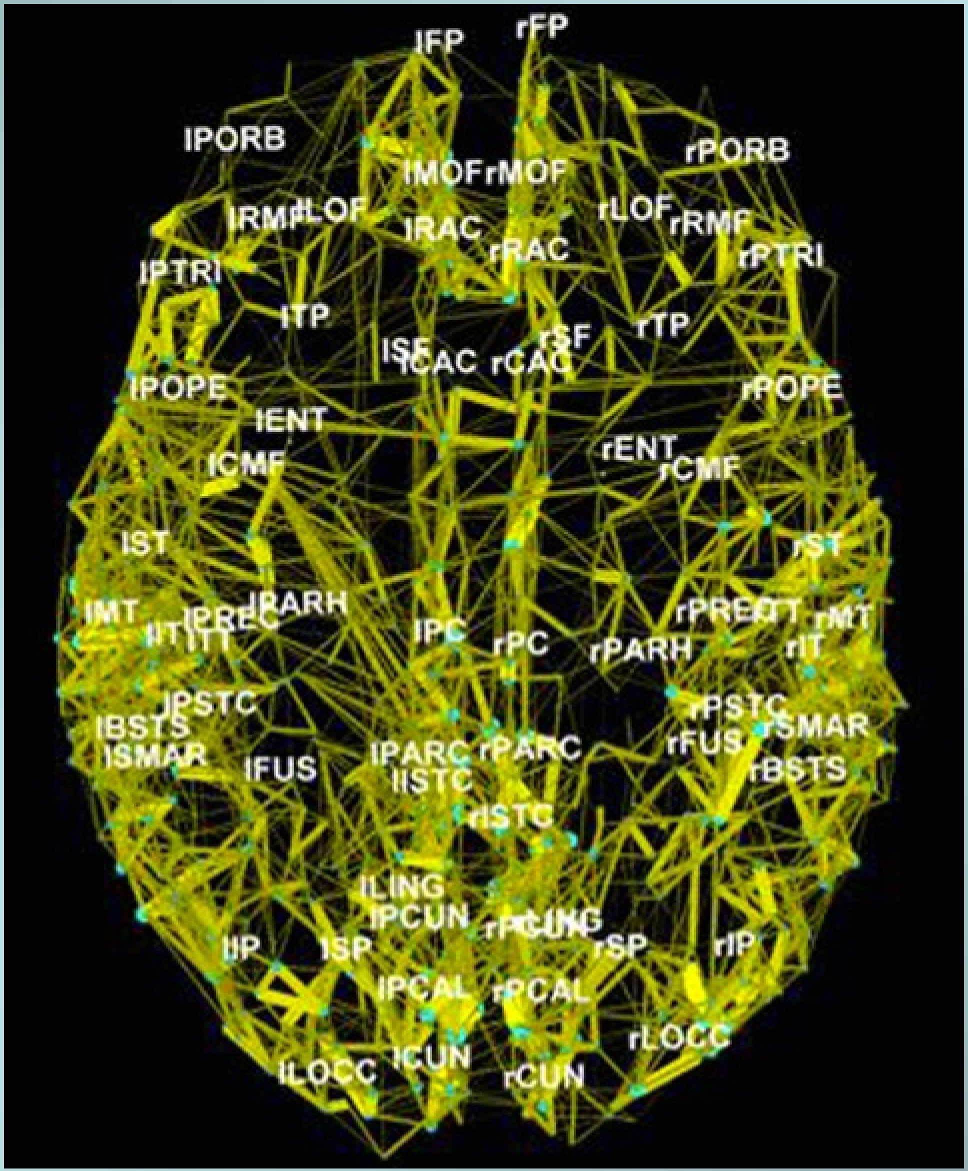
  length: 294 m, width: 33 m
- Cut one line of the power grid
- Black-out in

  Germany (> 10 Mio people)

  France (5 Mio people)

  Austria, Belgium, Italy, Spain

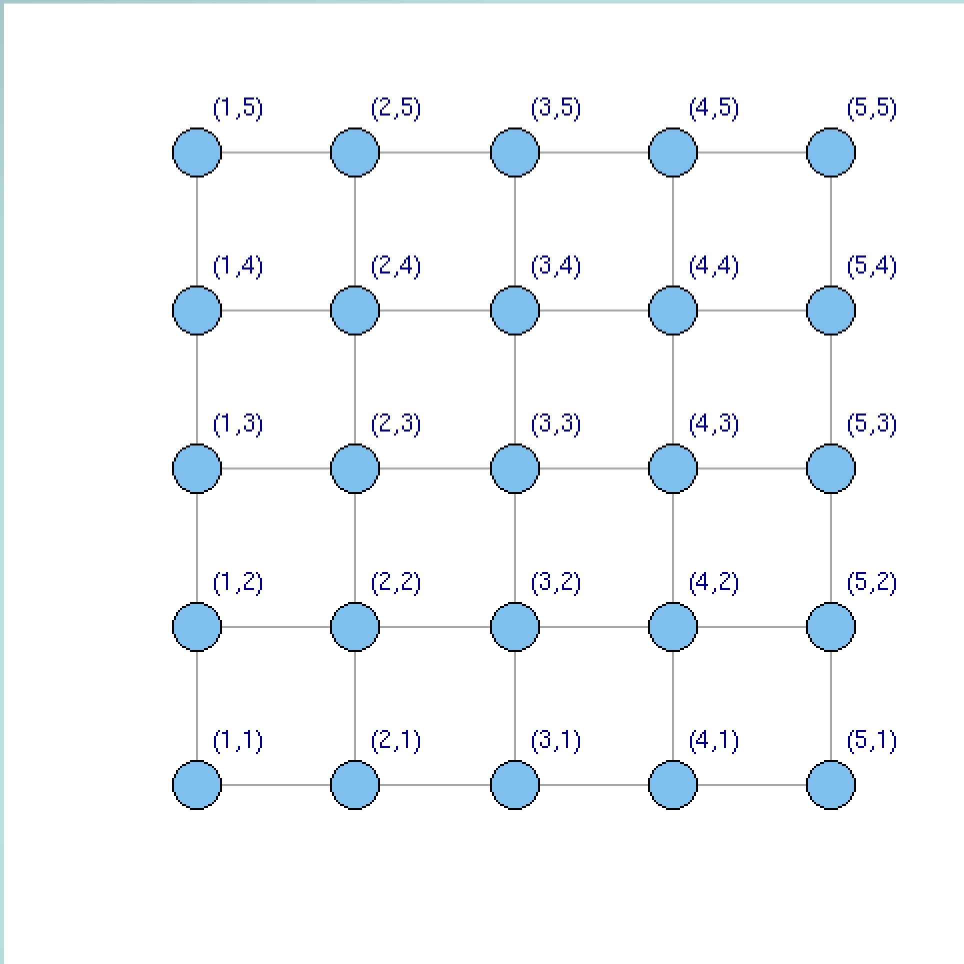
Cerebral cortex



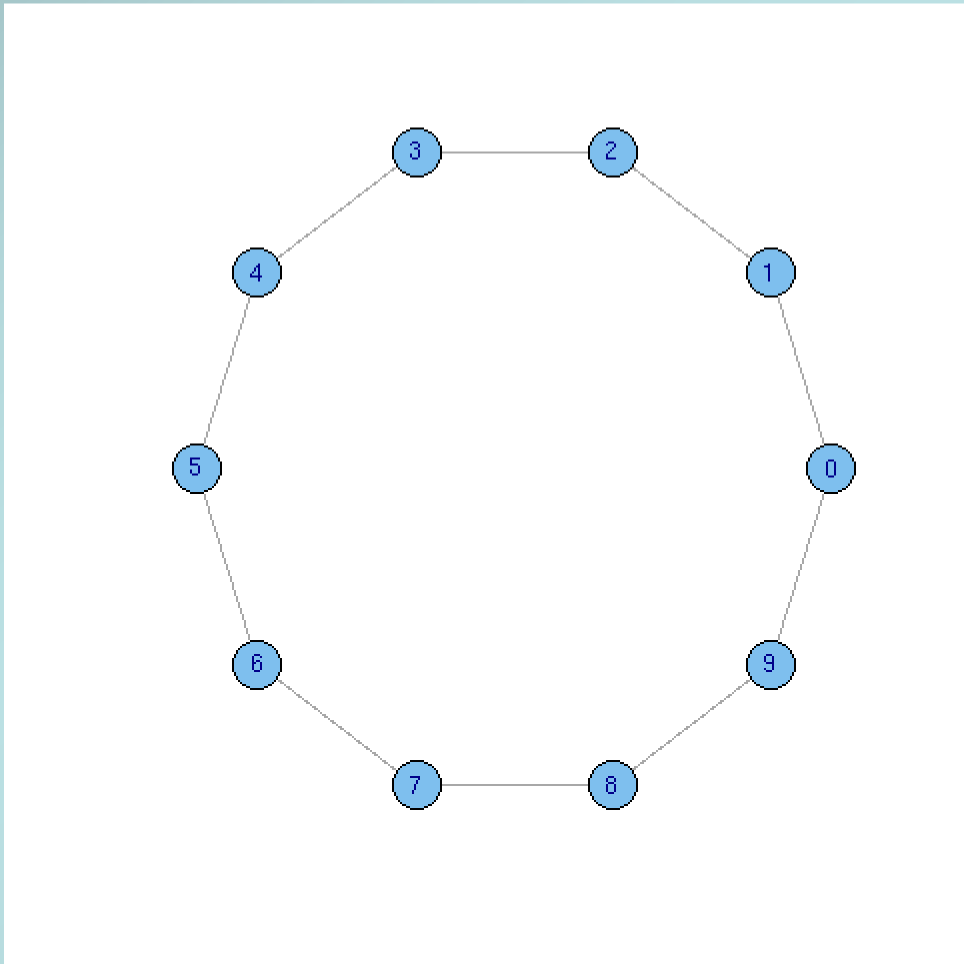
Useful approaches with networks

- **Immunization** problems (spreading of diseases)
- **Functioning** of biological/physiological processes as protein networks, brain dynamics, colonies of thermites and of social networks as network of vehicle traffic in a region, air traffic, or opinion formation etc.

Traditional networks in Physics lattices



Rings



Networks with Complex Topology

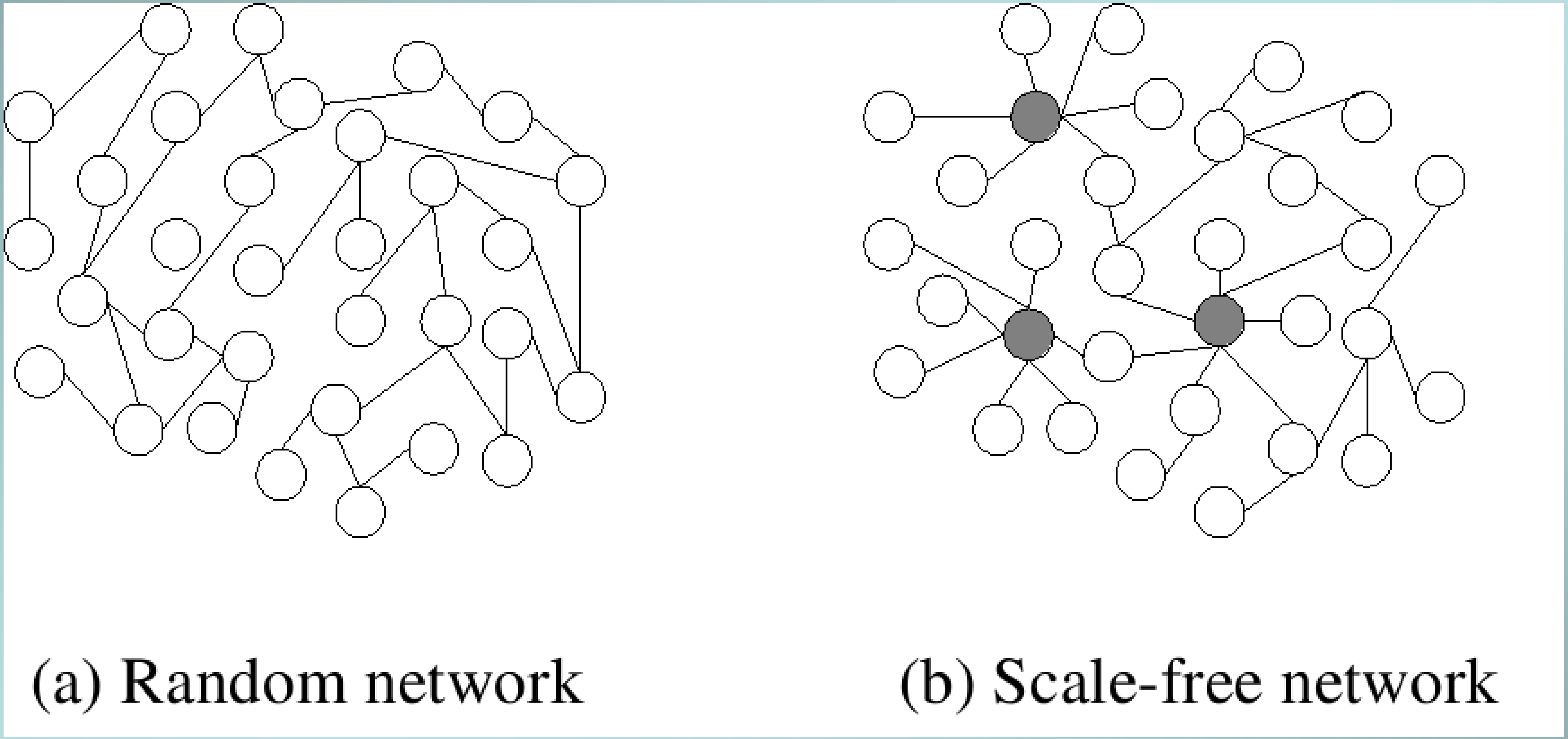
- Random graphs/networks (Erdös, Renyi, 1959)

- Small-world (SW) networks (Watts, Strogatz, 1998

  F. Karinthy hungarian writer – SW hypothesis, 1929)
- Scale-free networks (SF) (Barabasi, Albert, 1999;

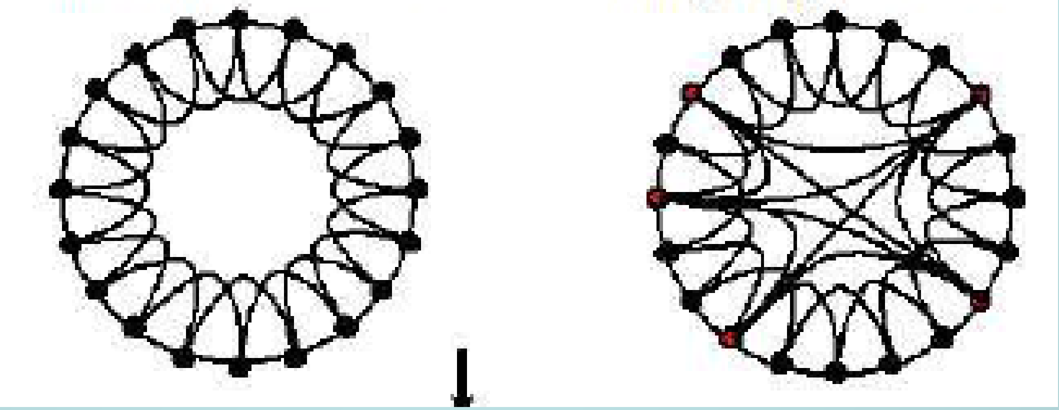
  D. de Solla Price – number of citations – heavy tail distribution, 1965)

Types of complex networks



fraction of nodes in the network having at least *k* connections to other nodes have a power law scaling – **existence of hubs**

Small-world Networks



Nearest neighbour Connections

Nearest neighbour and a few long-range connections (with probability p)

Regular ➔ Complex Topology

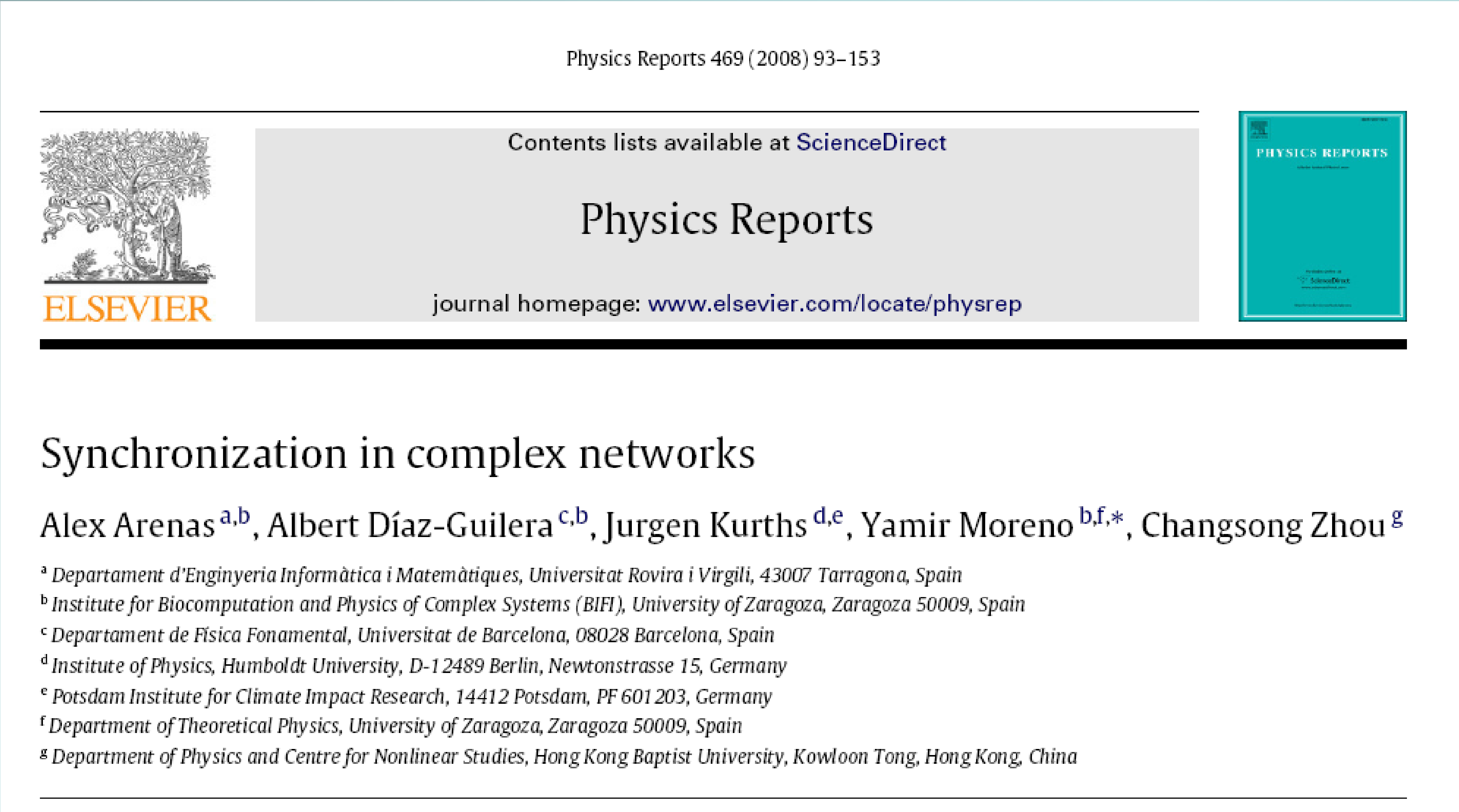
General description of a graph/complex network

- n nodes
- m edges
- $a_{i,j} = 1$ if node i and j are connected; 0 otherwise
- $A = ((a_{i,j}))$ adjacency matrix

- Network undirected $a_{i,j} = a_{j,i}$ symmetric characterized by eigenvalues and eigenvectors

- k_i - degree of node i, P(k) – degree distribution
- $K = \langle k_i \rangle$ mean degree

Dynamics on the nodes - synchronization



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Synchronization in complex networks

Alex Arenas[a,b], Albert Díaz-Guilera[c,b], Jurgen Kurths[d,e], Yamir Moreno[b,f,∗], Changsong Zhou[g]

[a] Departament d'Enginyeria Informàtica i Matemàtiques, Universitat Rovira i Virgili, 43007 Tarragona, Spain
[b] Institute for Biocomputation and Physics of Complex Systems (BIFI), University of Zaragoza, Zaragoza 50009, Spain
[c] Departament de Física Fonamental, Universitat de Barcelona, 08028 Barcelona, Spain
[d] Institute of Physics, Humboldt University, D-12489 Berlin, Newtonstrasse 15, Germany
[e] Potsdam Institute for Climate Impact Research, 14412 Potsdam, PF 601203, Germany
[f] Department of Theoretical Physics, University of Zaragoza, Zaragoza 50009, Spain
[g] Department of Physics and Centre for Nonlinear Studies, Hong Kong Baptist University, Kowloon Tong, Hong Kong, China

Universality in the synchronization of weighted random networks

Our intention:

What is the influence of weighted coupling for complete synchronization

Weighted Network of N Identical Oscillators

$$\begin{aligned}\dot{\mathbf{x}}_i &= \mathbf{F}(\mathbf{x}_i) + \sigma \sum_{j=1}^{N} W_{ij} A_{ij} [\mathbf{H}(\mathbf{x}_j) - \mathbf{H}(\mathbf{x}_i)], \\ &= \mathbf{F}(\mathbf{x}_i) - \sigma \sum_{j=1}^{N} G_{ij} \mathbf{H}(\mathbf{x}_j), \quad i = 1, \ldots, N,\end{aligned}$$

F – dynamics of each oscillator

H – output function

G – coupling matrix combining adjacency A and weight W

$G_{ij} = -W_{ij}$ for $i \neq j$ $G_{ii} = \sum_j W_{ij} A_{ij} = S_i$

S_i - intensity of node i (includes topology and weights)

General Condition for Synchronizability

Stability of synchronized state

$$\{\mathbf{x}_i = \mathbf{s}, \forall\, i \,|\, \dot{\mathbf{s}} = \mathbf{F}(\mathbf{s})\}$$

N eigenmodes of

$$\dot{\xi}_i = [D\mathbf{F}(\mathbf{s}) - \sigma\lambda_i D\mathbf{H}(\mathbf{s})]\xi_i,$$

λ_i *i*th eigenvalue of G

Main results

Synchronizability universally determined by:

- mean degree K and

- heterogeneity of the intensities

$$\frac{S_{\max}}{S_{\min}} \quad \text{or} \quad \frac{\Omega}{S_{\min}}$$

$S_{\min}, \ S_{\max}$ - minimum/ maximum intensities

Synchronizability – Master Stability Formalism

(Pecora&Carrol (1998)

Synchronizability Ratio $R = \lambda_{\max}/\lambda_{\min}$

Stability Interval $I_s = (\alpha_1/\lambda_{\min}, \alpha_2/\lambda_{\max})$

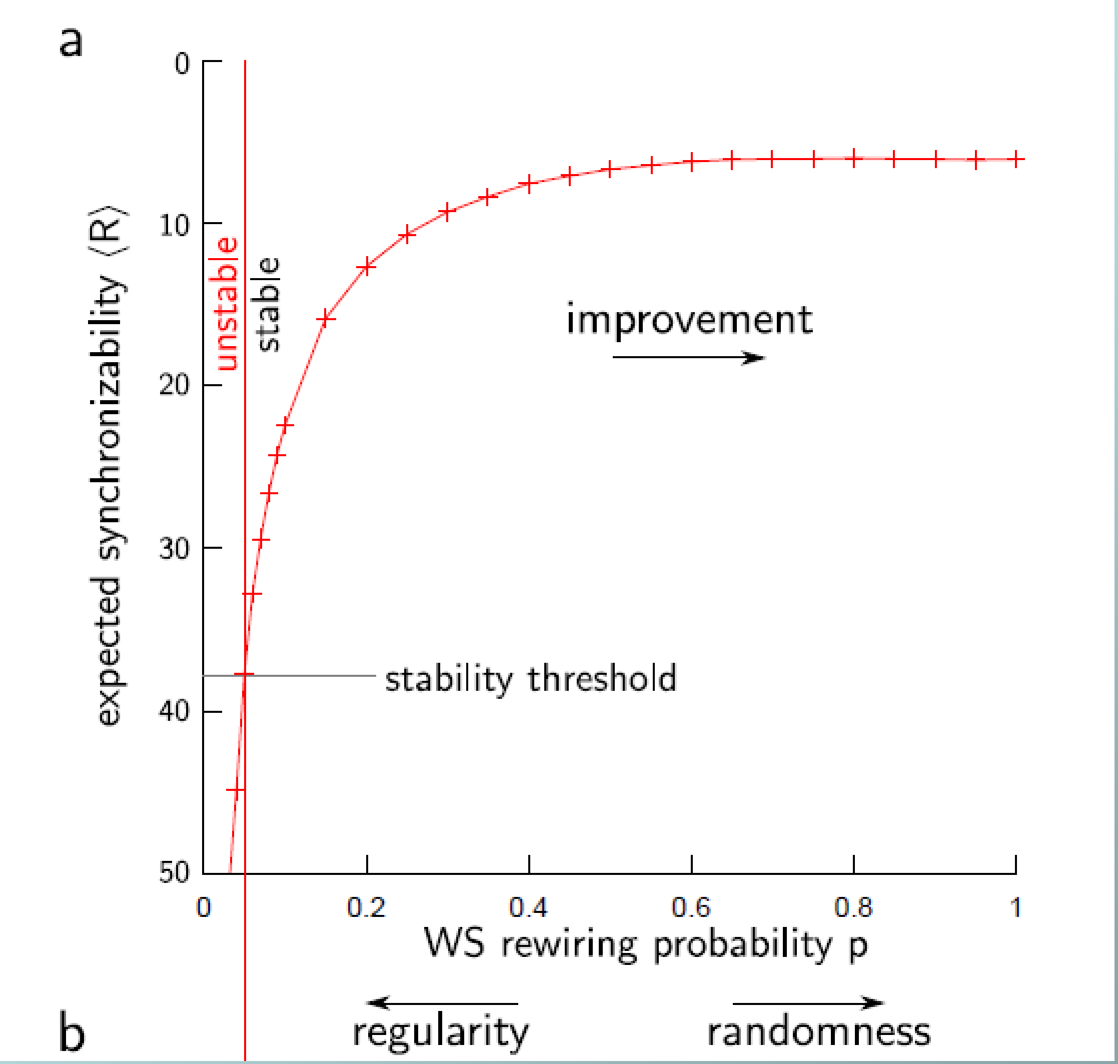
Synchronizability condition $R < \alpha_2/\alpha_1$

Puzzle: best synchronizable = most random

However:
Most **real-world** networks far from being random

How to understand?

So far: **local stability** studied

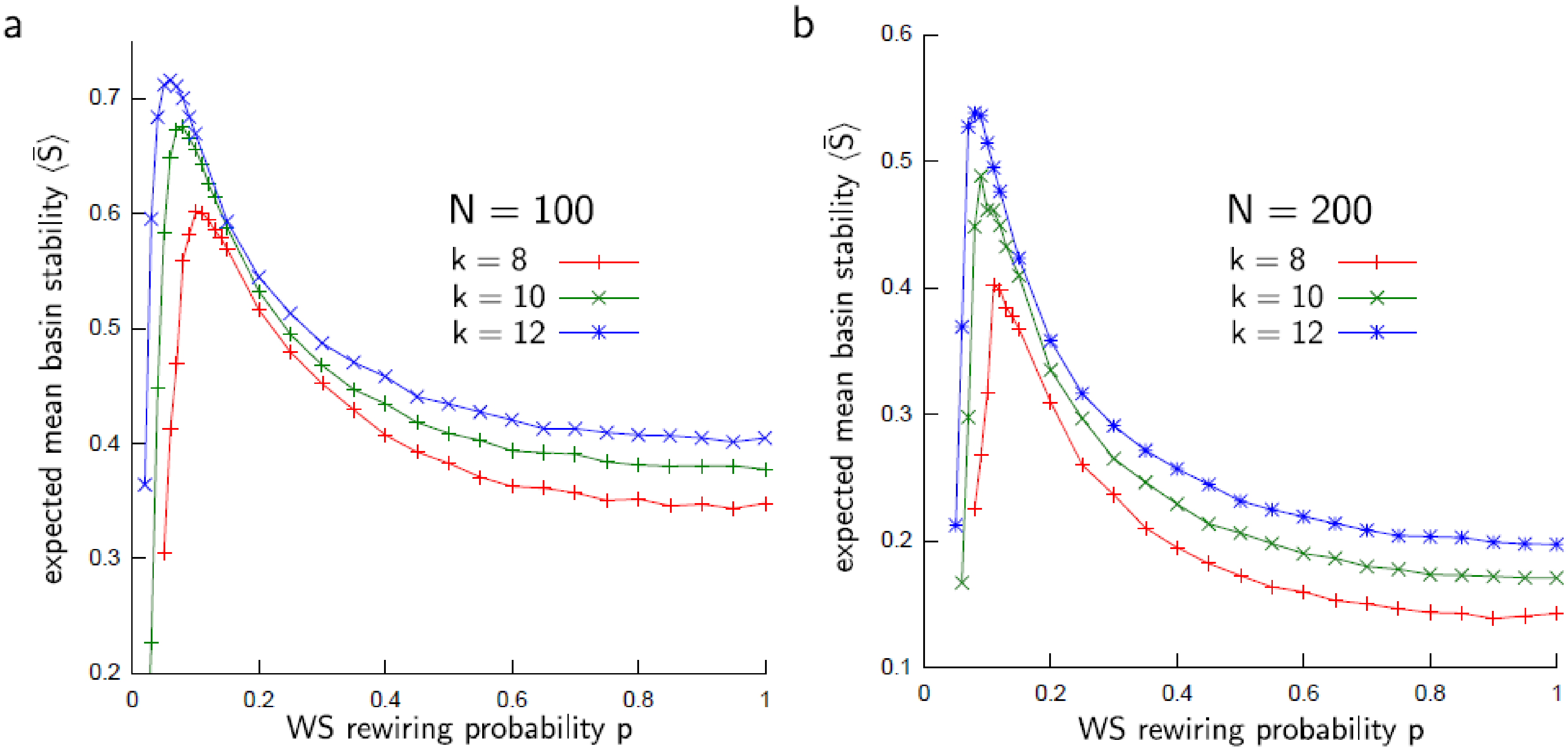


Basin Stability

basin volume of a state (regime)

measures likelihood of arrival at this state (regime)

NATURE PHYSICS (in press)

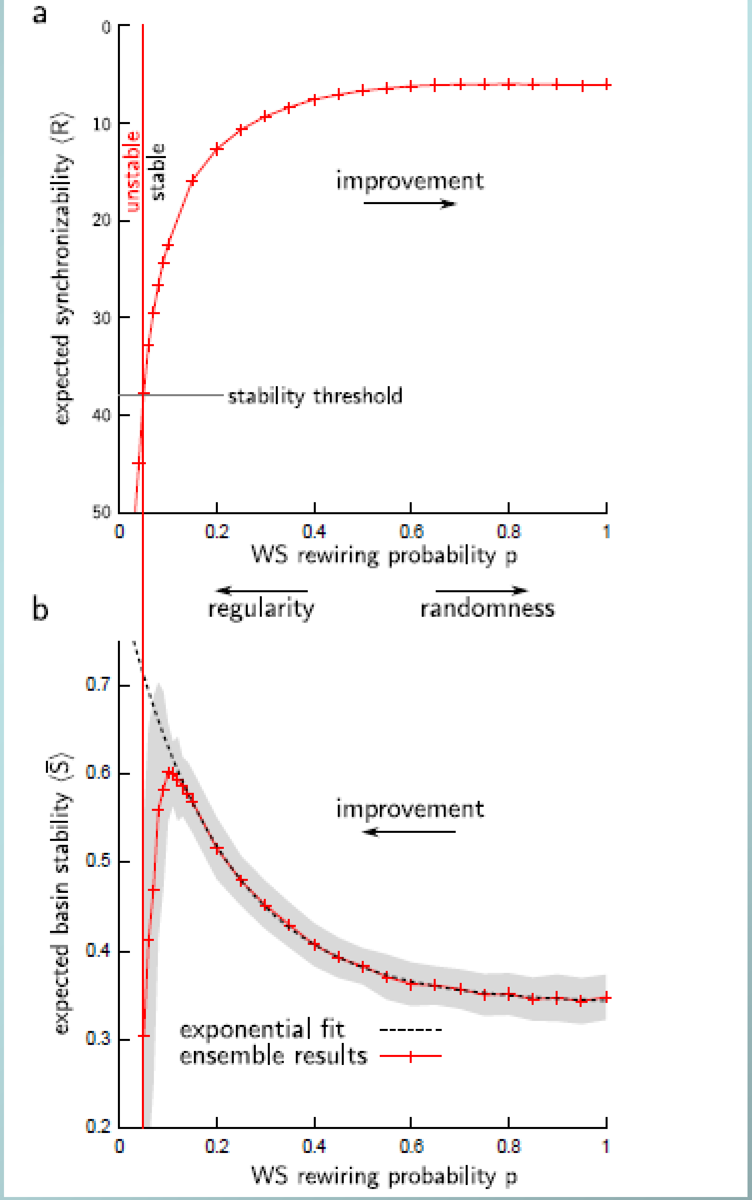


Supplementary Figure S1: **Basin Stability in Rössler networks.** Expected basin stability $\langle \bar{S} \rangle$ versus p. The grey shade indicates $\pm$ one standard deviation. The dashed line shows an exponential fitted to the ensemble results for $p \geq 0.15$. Solid lines are guides to the eye. **a**: $N = 100$, **b**: N=200.

Synchronizability and basin stability inWatts-Strogatz (WS) networks
of chaotic oscillators.

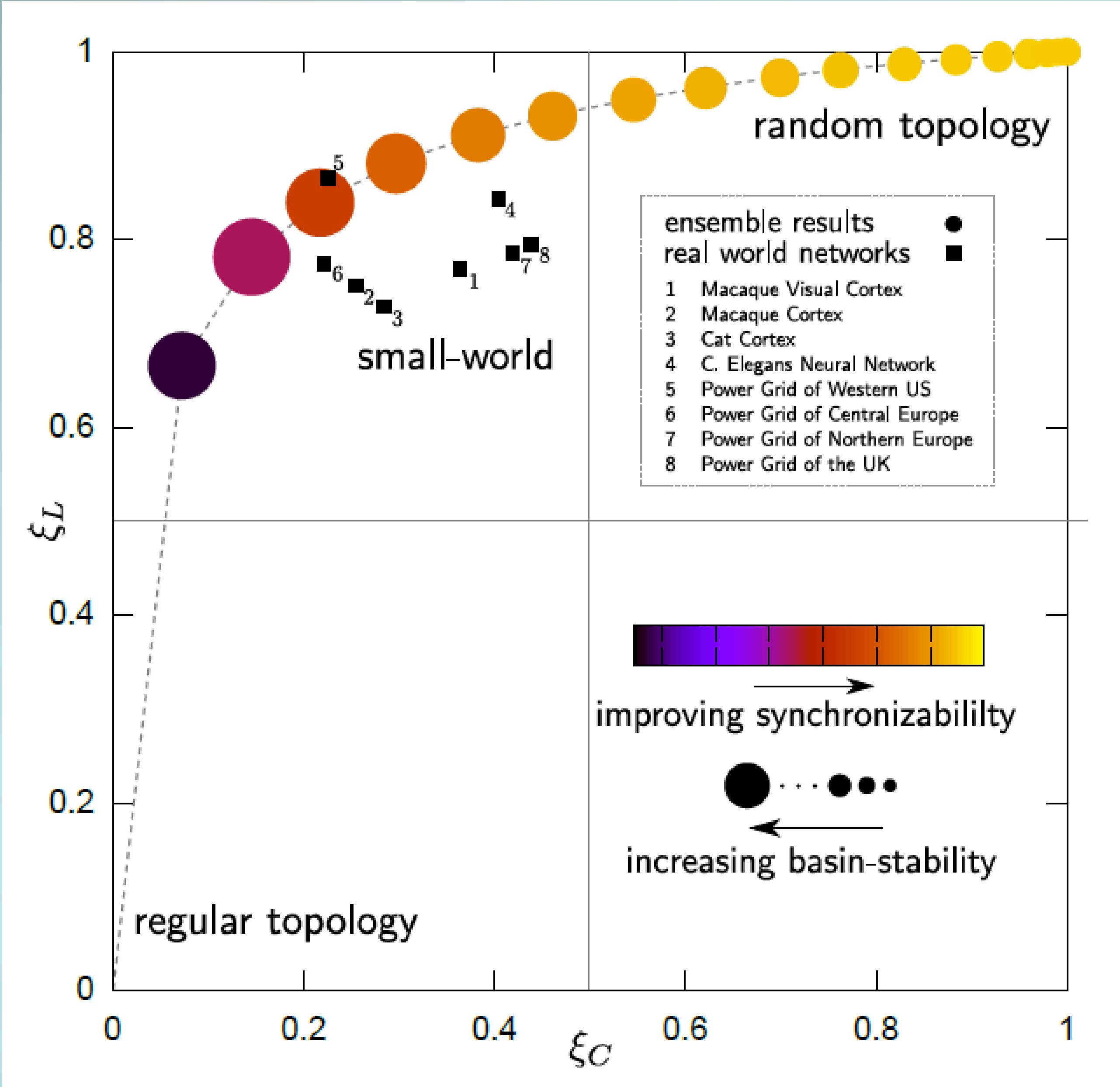
a: Expected synchronizability R versus the WS model's parameter p.
The scale of the y-axis was reversed to indicate improvement upon increase in p.

b: Expected basin stability S versus p. The grey shade indicates one standard deviation.
The dashed line shows an exponential fitted to the ensemble results for p > 0.15.
Solid lines are guides to the eye. The plots shown were obtained for N = 100 oscillators of Roessler type, each having on average k = 8 neighbours. Choices of larger N and different k produce results that are qualitatively the same.



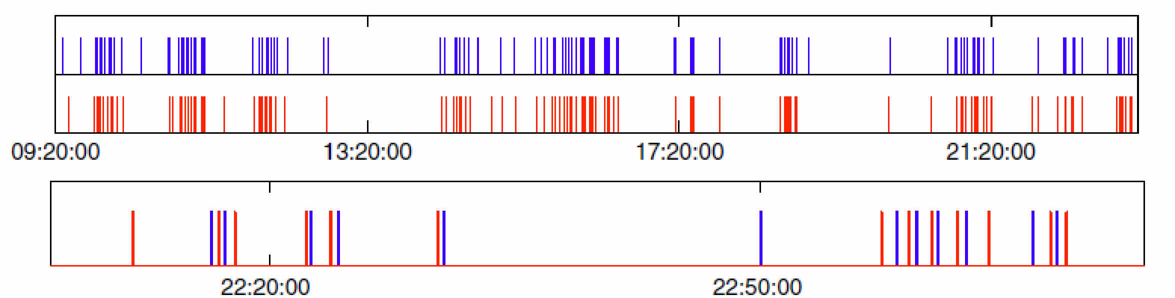
Topological comparison of ensemble results with real-world networks.

- Circle represents the results for Watts-Strogatz networks with N = 100, k = 10 and rewiring probability p (increasing from left to right 0.05…1.0).
- Circle's area proportional to expected basin stability S.
- Circle's colour indicates expected synchronizability R.
- Squares represent real-world networks reported to display a small-world topology.



Social Networks: Human Communication

- Short Message correspondence (SMS)
- Data record: 1.5 Mio SMS (times sent to whom)



- User A (blue) and B (red) –

  sending-response pattern

Statistics

- 50 % of users sent > 90 % of messages to ONE partner

  ➔ concentrate on pairs of users
- Distribution of the inter-event time P(t) bimodal:

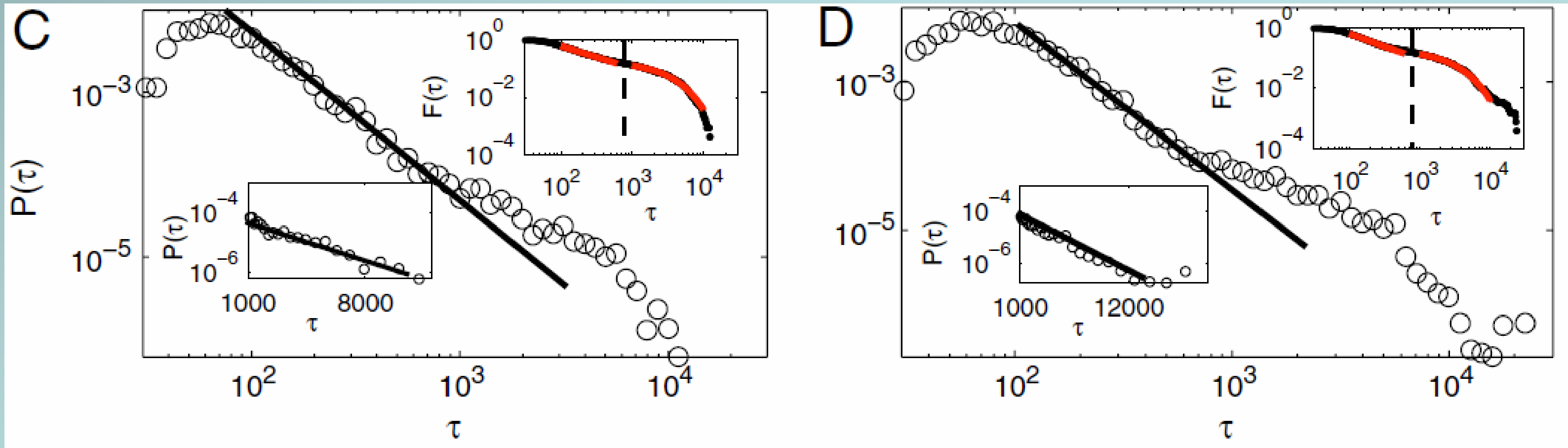
  power law on small scale (2 – 20 min)

  exponential tail (20 min – 6 hours)

$$P(\tau) = \begin{cases} \tau^{-\gamma}, & \tau < \tau_0 \\ e^{-\beta\tau}, & \tau > \tau_0 \end{cases}$$

- Not a finite-size effects (no power-law heavy tails)

Distribution of inter-event time (one person sent)



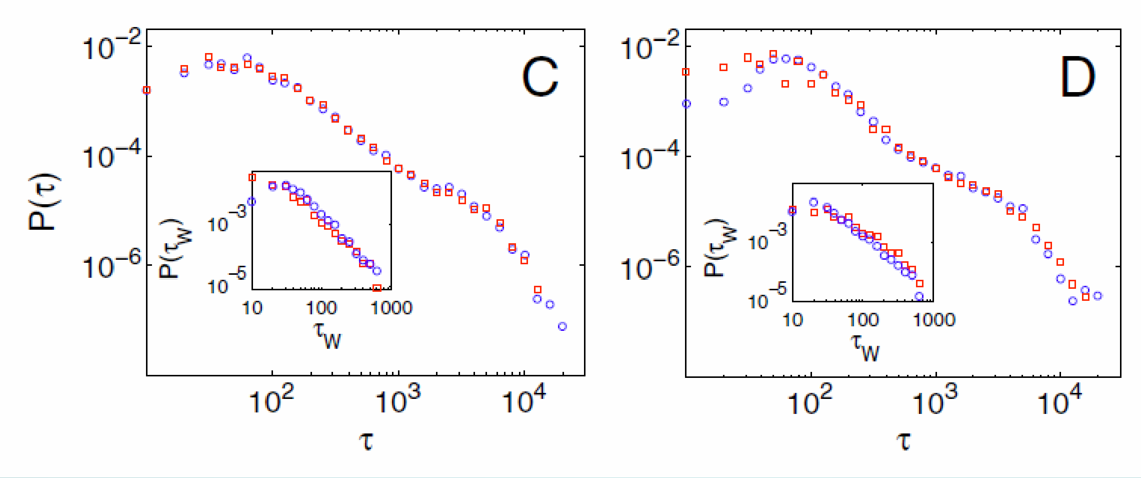
Phenomenon and modeling

- Typical sequence:

  Burst initiated by one user (exponential)

  Followed by a frequent mutual communication (power law)
- Model:

  Interacting priority queues of two persons
- Parameters: processing time (adding/removal of tasks), random initiation of tasks, interaction probability, waiting time

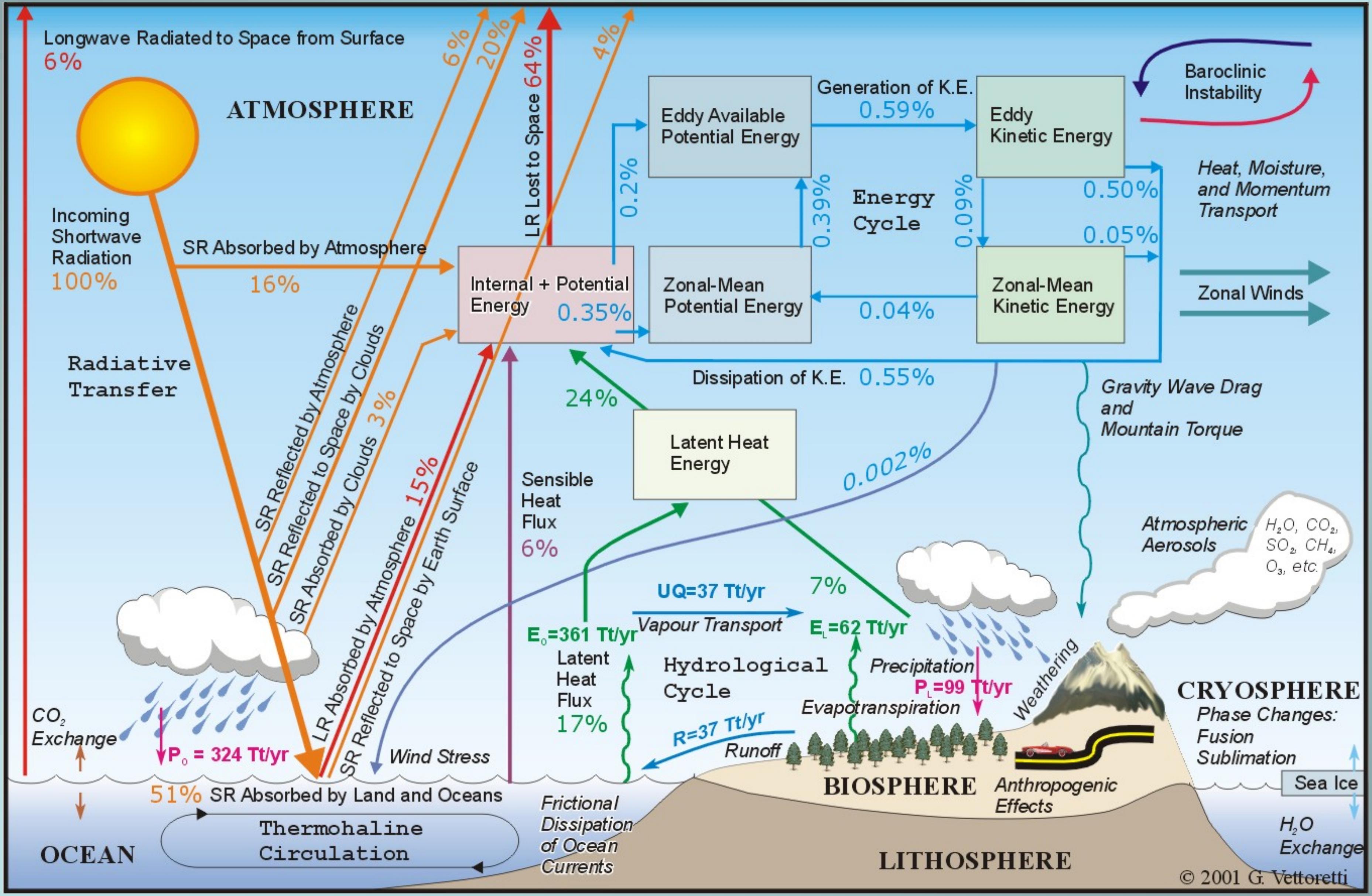


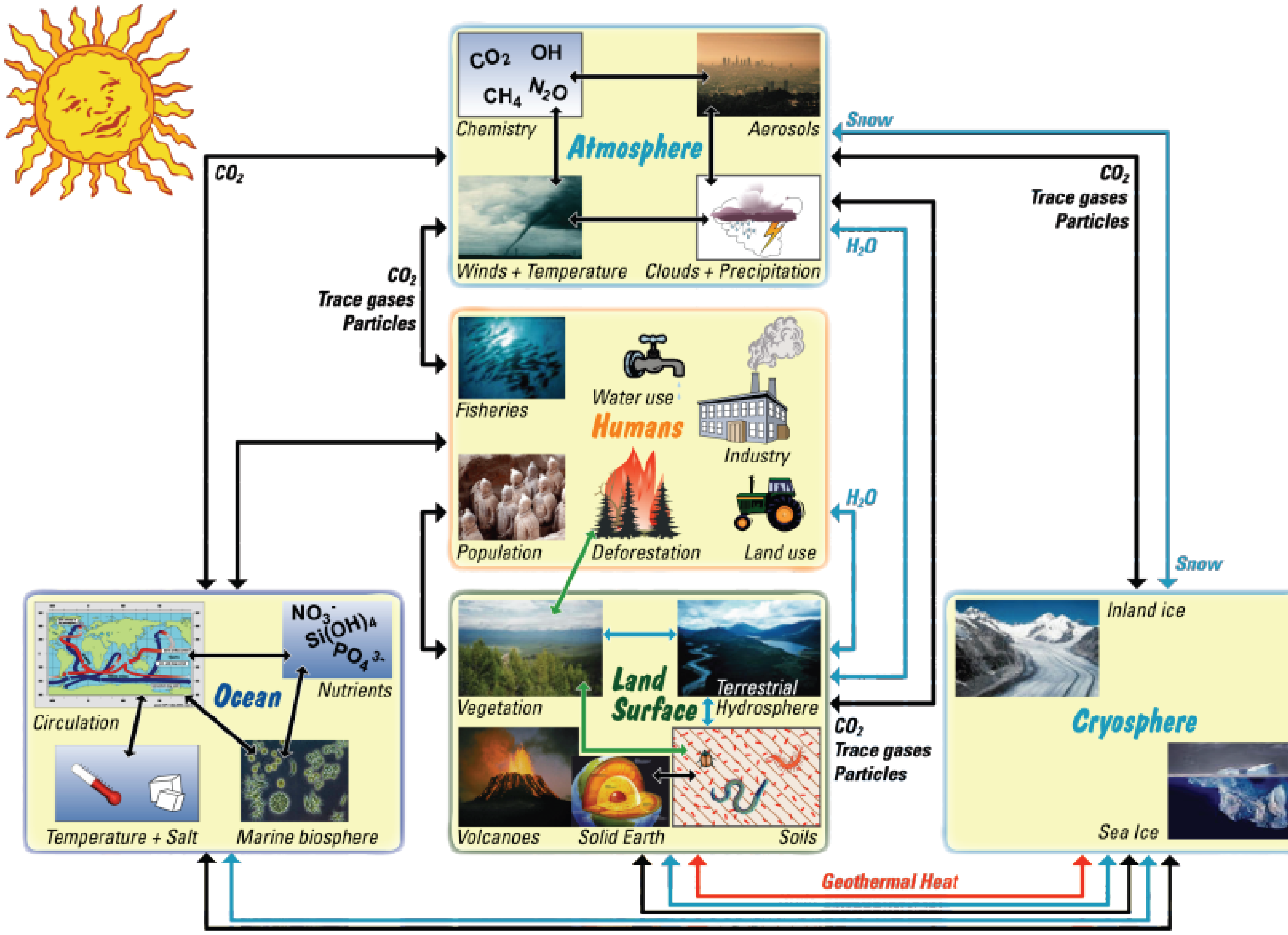
empirical data (blue) vs. fitted model (red)

PNAS 107, 18803 (2010)

System Earth

System Earth - Horrendogram




Atmosphere
CO_2 OH CH_4 N_2O
Chemistry
Aerosols
Winds + Temperature
Clouds + Precipitation
Snow
CO_2
CO_2
Trace gases
Particles
H_2O
CO_2
Trace gases
Particles
Humans
Fisheries
Water use
Industry
Population
Deforestation
Land use
H_2O
Snow
Ocean
NO_3^- $Si(OH)_4$ PO_4^{3-}
Nutrients
Circulation
Temperature + Salt
Marine biosphere
Land Surface
Vegetation
Terrestrial Hydrosphere
Volcanoes
Solid Earth
Soils
CO_2
Trace gases
Particles
Cryosphere
Inland ice
Sea Ice
Geothermal Heat

System Earth

Subsystem: Climate

Network Reconstruction from a continuous dynamic system (structure vs. functionality)

New (inverse) problems arise!

Is there a backbone underlying the climate system?

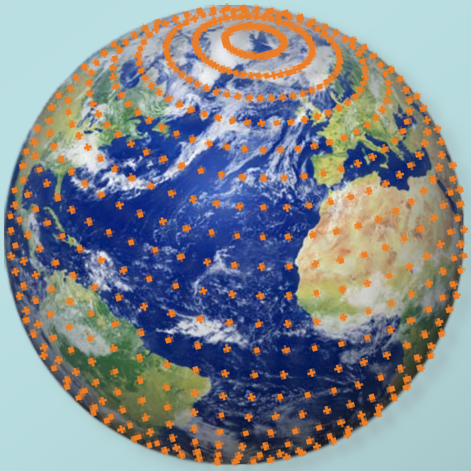
1st Step 2D:

Analysis of Earth´s Surface - Temperature Data

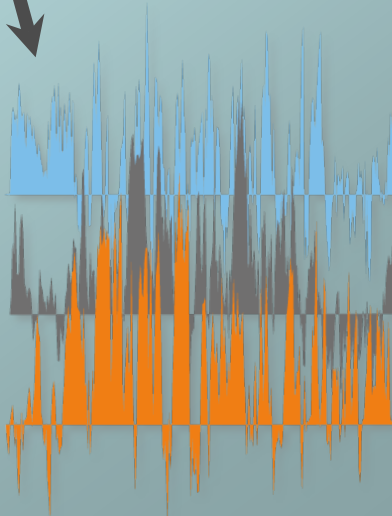
Climate Networks

Earth system

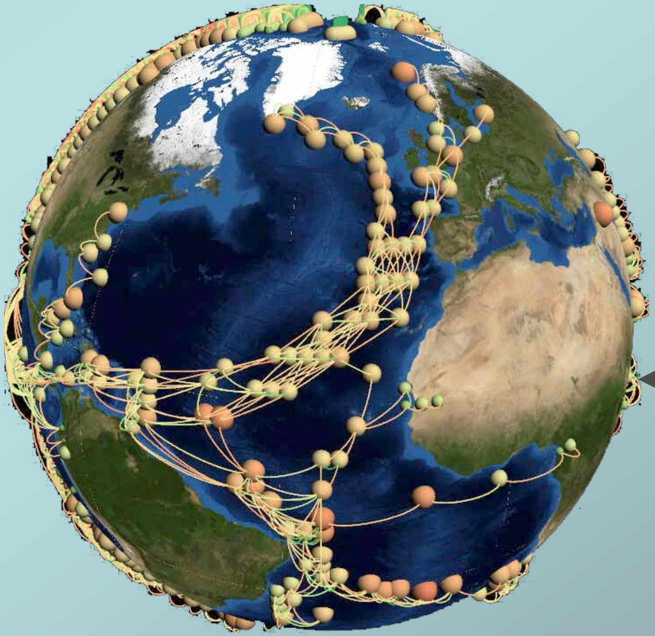
Observation sites



Time series

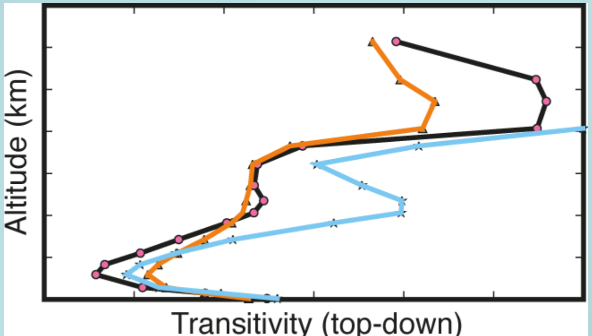


Climate network



Network analysis

$$C_v^* = \frac{W^2 \langle a^+_{vi} a^+_{ij} a^+_{jv} \rangle^w_{ij}}{k_v^{*2}}$$



Betweenness

Betweenness Centrality B

Number of shortest paths that connect nodes j and k n_{jk}

Number of shortest paths that connect nodes j and k AND path through node i $n_{jk}(i)$

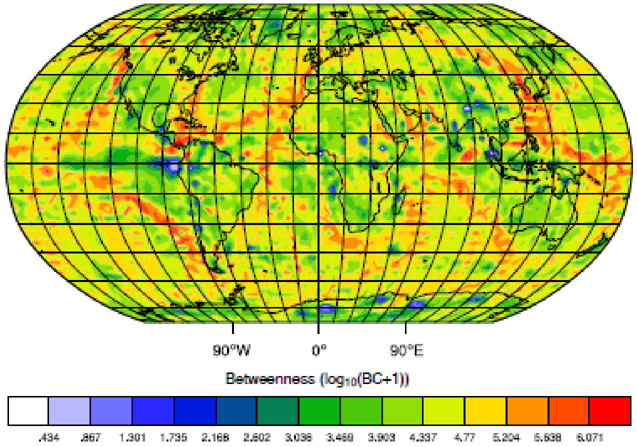
Local betweenness of node i

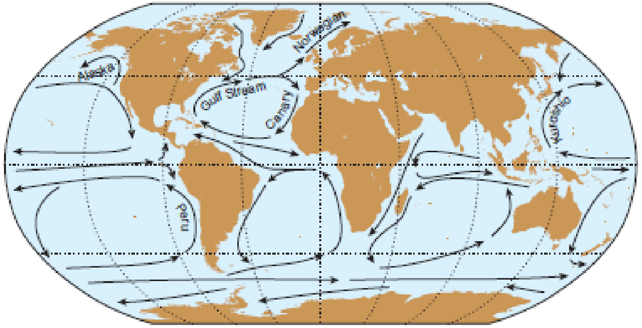
$$b_i = \sum_{j,k,j\neq k} \frac{n_{jk}(i)}{n_{jk}}$$

(local and global aspects included!)

Betweenness Centrality B = < b_i >

Backbone structures → surface ocean currents?



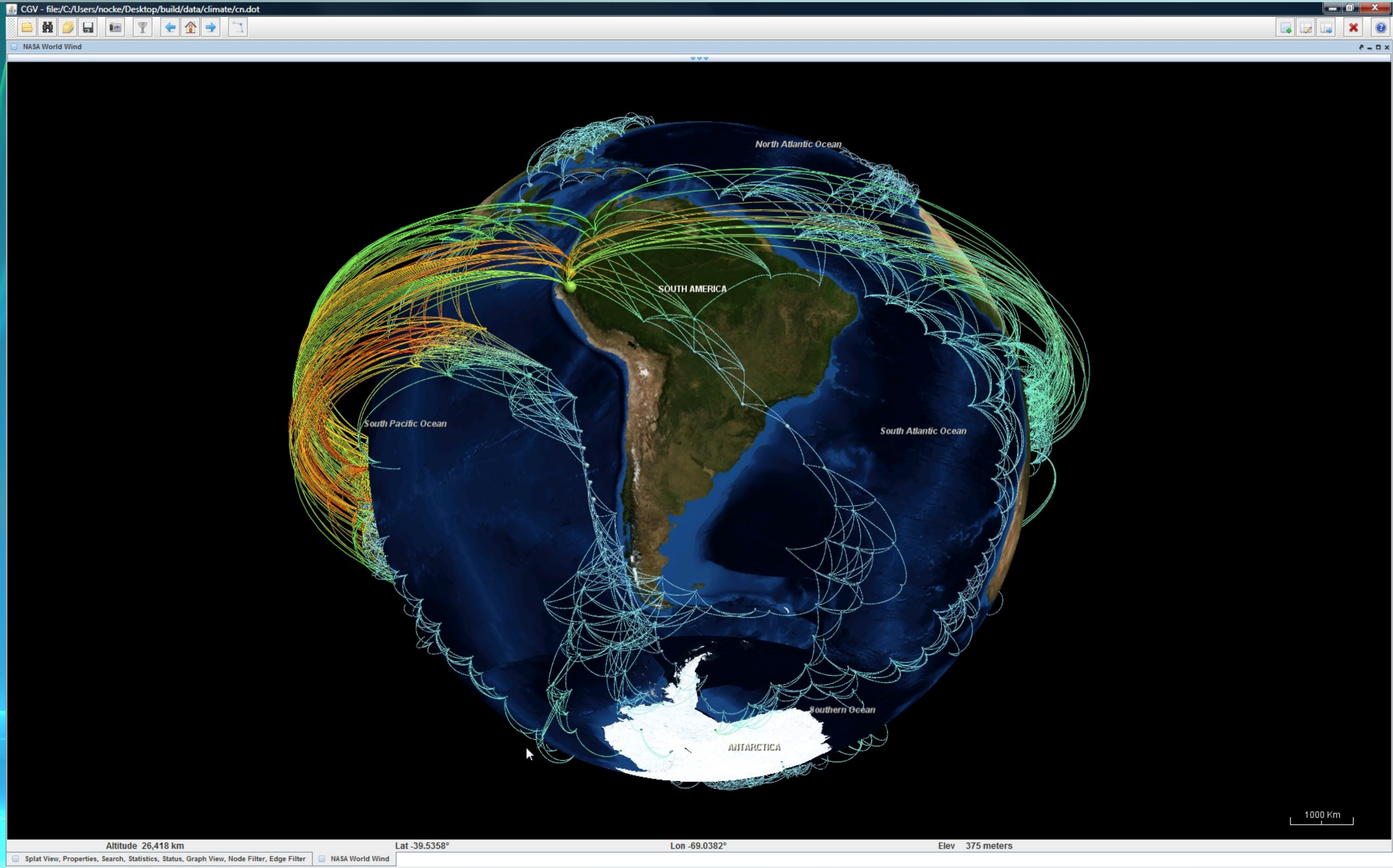


Donges et al., Europhys Lett **87**, 48007 (2009)

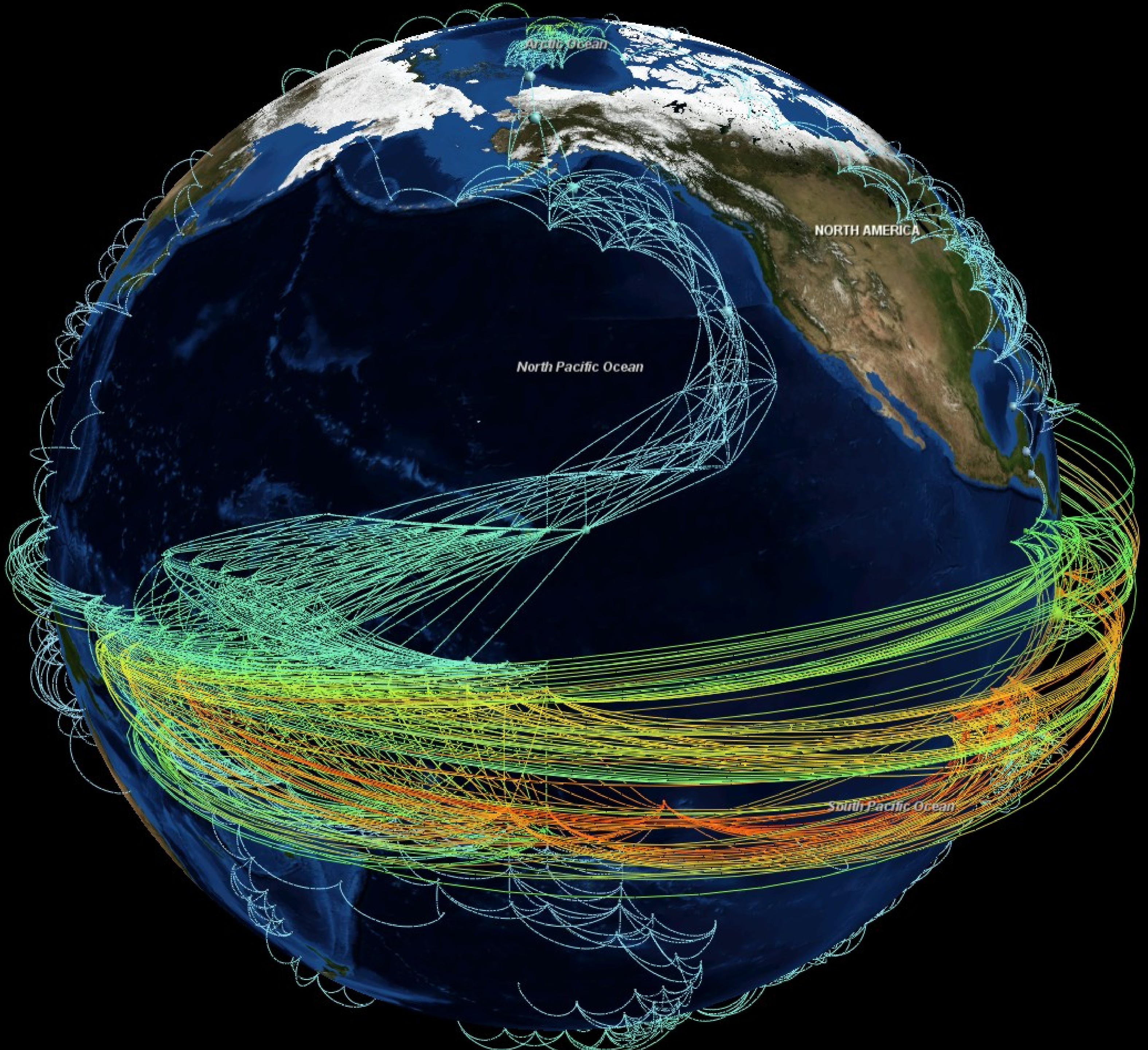
Infer long-range connections –

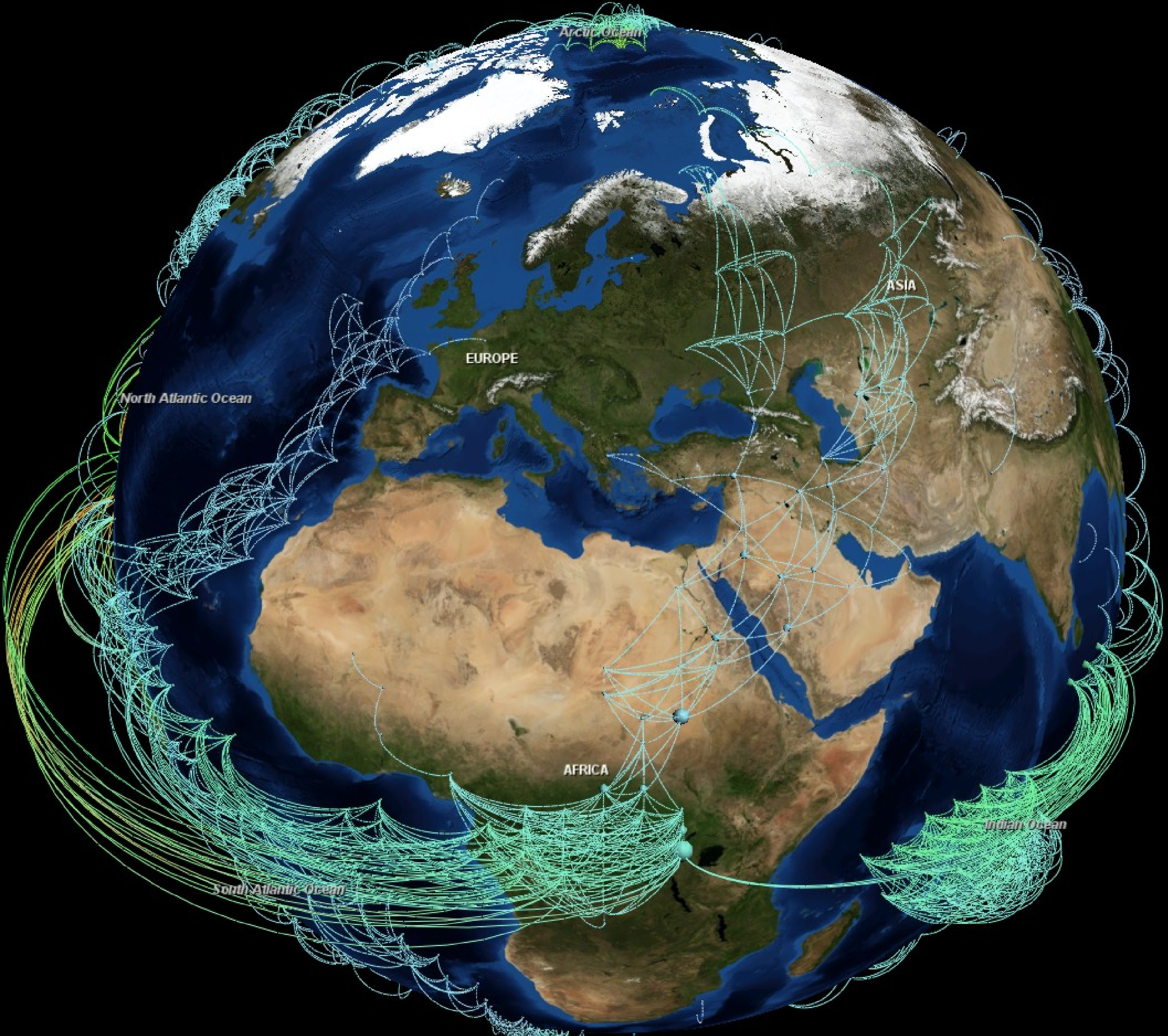
Teleconnections

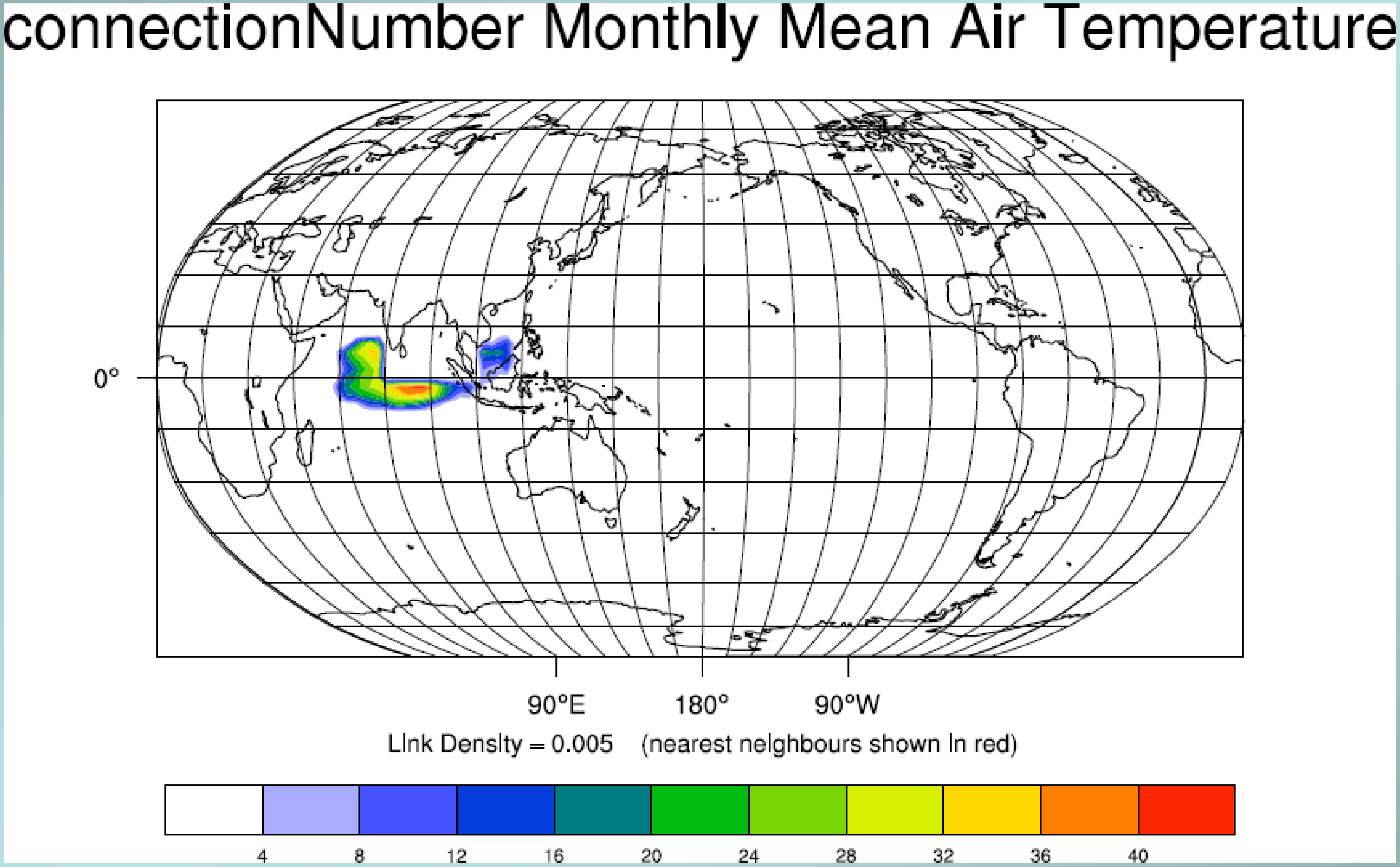
Complex network approach to climate system



Altitude 26,418 km
Lat -39.5358°
Lon -69.0382°
Elev 375 meters


Arctic Ocean
NORTH AMERICA
North Pacific Ocean
South Pacific Ocean


Arctic Ocean
ASIA
EUROPE
North Atlantic Ocean
AFRICA
Indian Ocean
South Atlantic Ocean


connectionNumber Monthly Mean Air Temperature
0°
90°E
180°
90°W
Link Density = 0.005 (nearest neighbours shown in red)
4
8
12
16
20
24
28
32
36
40

Earth is 3D
➔ 2nd Step:

Include Different Layers of the Atmosphere

Analysing the vertical dynamical structure of the Earth´s atmosphere

- Data: pressure data at 17 different geopotential heights
- Challenge: how to describe such interacting networks? –

  Network of Networks

  (originally used for brain modeling, PRL 2006) – but here different approach

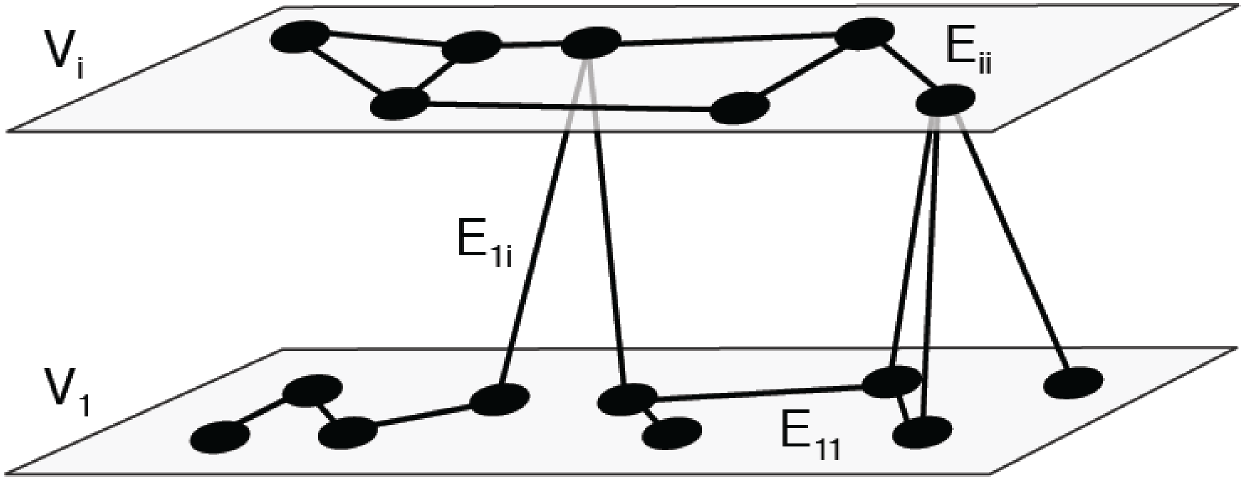
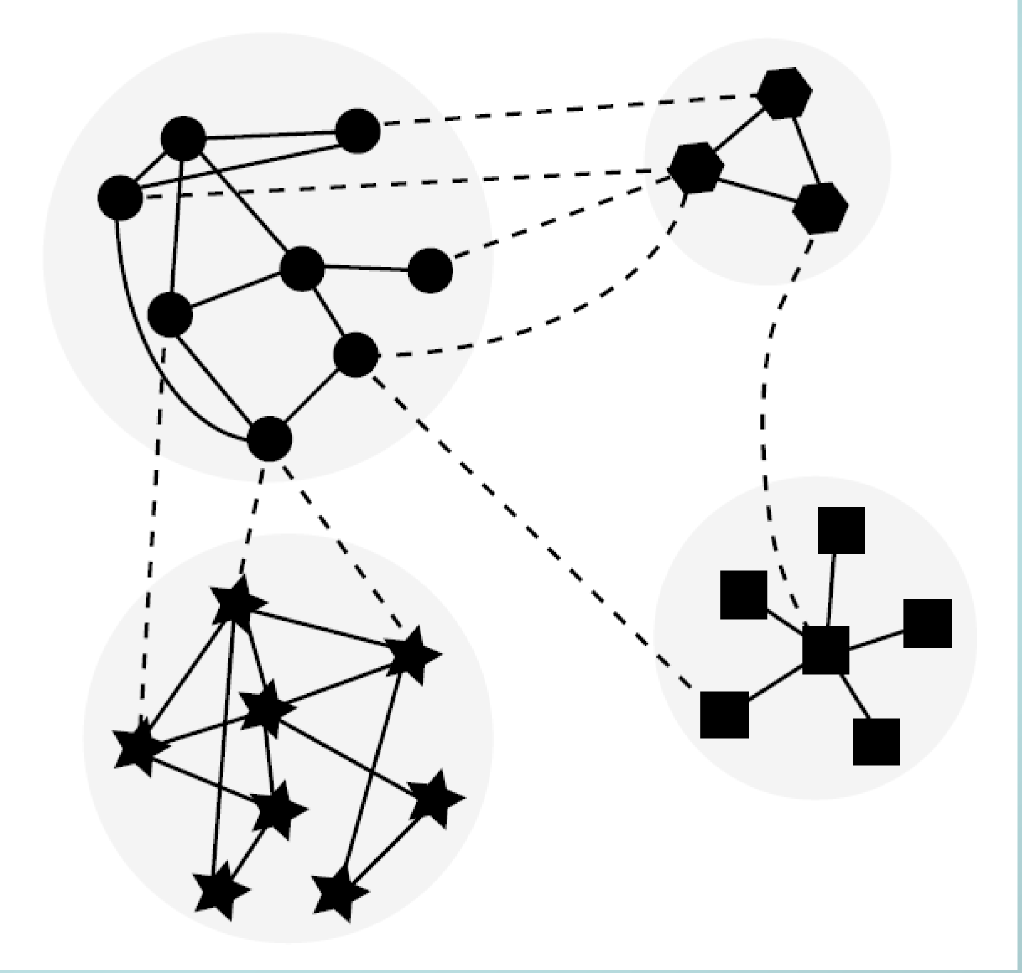


Fig. 3. Illustration of a coupled climate subnetwork as it is constructed in this work, where V_1 denotes the set of vertices in the near ground subnetwork and V_i that of another isobaric surface higher up in the atmosphere. E_{11}, E_{ii} are sets of internal edges of the two isobaric surfaces or subnetworks describing the statistical relationships within each isobaric surface, while E_{1i} contains information on their mutual statistical interdependencies.

Network of networks



-Links inside a subnetwork

- Links between different subnetworks

-New measures: cross-degree, cross-clustering, cross-pathways…

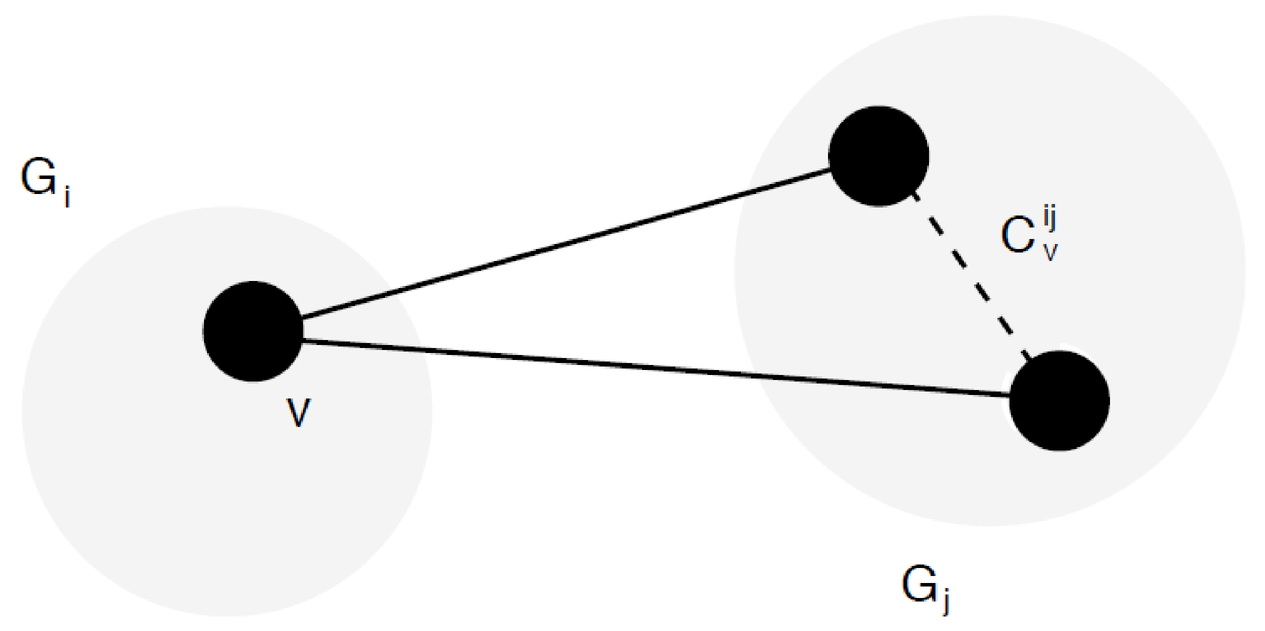
Modified Measures

- **Cross-average path length L_{ij}** – average length of shortest paths between two subnetworks G_i and G_j

$$\mathcal{L}_{ij} = \frac{1}{N_i N_j - M_{ij}} \sum_{v \in V_i, q \in V_j} d_{vq}$$

- **Cross-degree centrality k_v^{ij}** – number of neighbours a node v, which is in G_i, has in G_j

Cross-clustering coefficient C_v^{ij} – frequency that two randomly drawn neighbours in G_j of node v, which is in G_i, are also neighbours



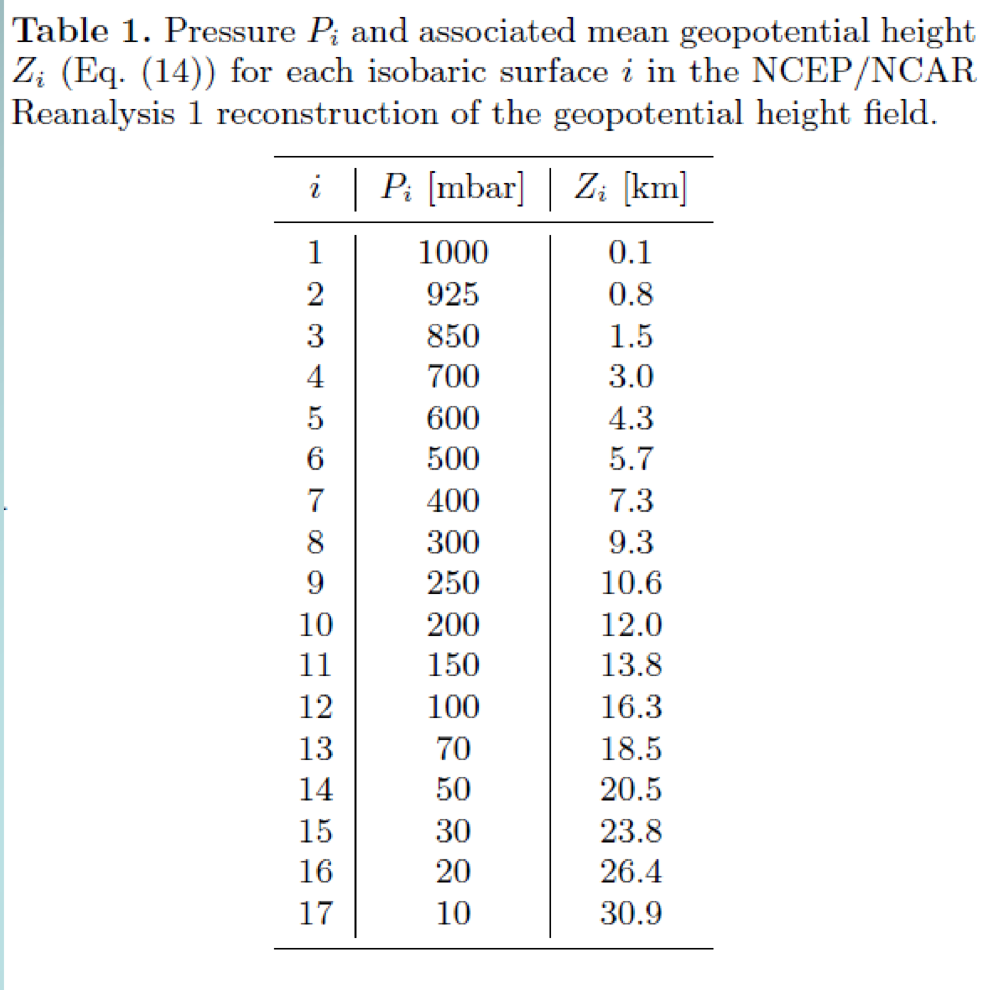


Table 1. Pressure P_i and associated mean geopotential height Z_i (Eq. (14)) for each isobaric surface i in the NCEP/NCAR Reanalysis 1 reconstruction of the geopotential height field.

i	P_i [mbar]	Z_i [km]
1	1000	0.1
2	925	0.8
3	850	1.5
4	700	3.0
5	600	4.3
6	500	5.7
7	400	7.3
8	300	9.3
9	250	10.6
10	200	12.0
11	150	13.8
12	100	16.3
13	70	18.5
14	50	20.5
15	30	23.8
16	20	26.4
17	10	30.9

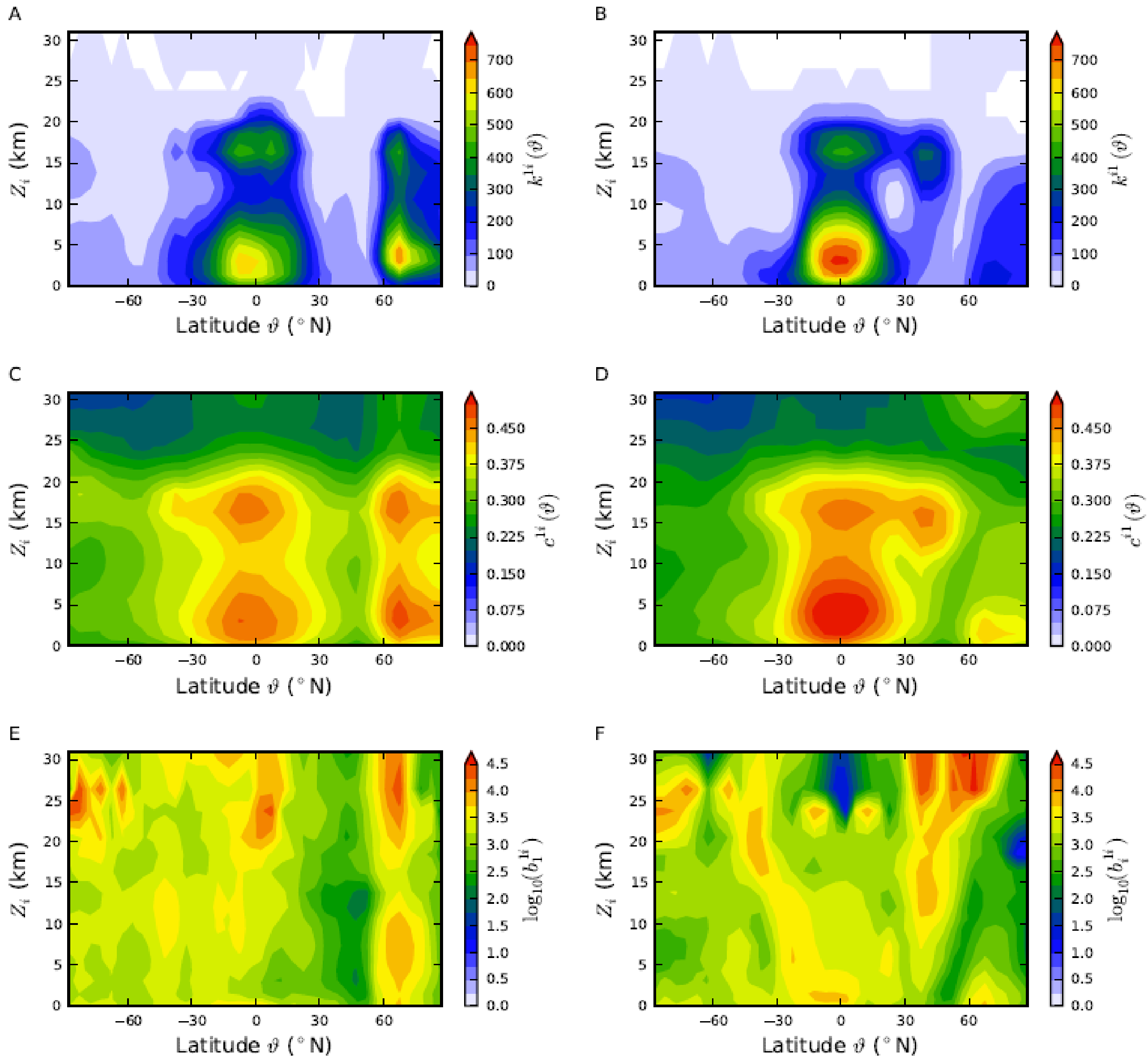


Fig. 7. Zonally averaged cross-degree centralities (A) $k^{1i}(\vartheta)$ pointing "upwards" from the near ground level 1 to all other isobaric surfaces i and (B) $k^{i1}(\vartheta)$ projecting "downwards", zonally averaged cross-closeness centralities (C) $c^{1i}(\vartheta)$ pointing "upwards" and (D) $c^{i1}(\vartheta)$ projecting "downwards", (E) $b_1^{1i}(\vartheta)$ near ground and (F) $b_i^{1i}(\vartheta)$ upper level component of zonally averaged cross-betweenness centrality for a threshold of $T = 0.4$. Panel (B) can be interpreted to show the number of cross-edges connecting a certain volume element with the whole near ground isobaric surface, averaged along bands of approximately equal latitude (approximately because of the geodesic grid).

Network analysis to identify large scale atmospheric motions and as a visualisation tool

Asian Summer Monsoon

Influence
Indian Summer Monsoon (ISM)
on
Eastasian Summer Monsoon
(EASM)

Clim. Dynamics (2012)

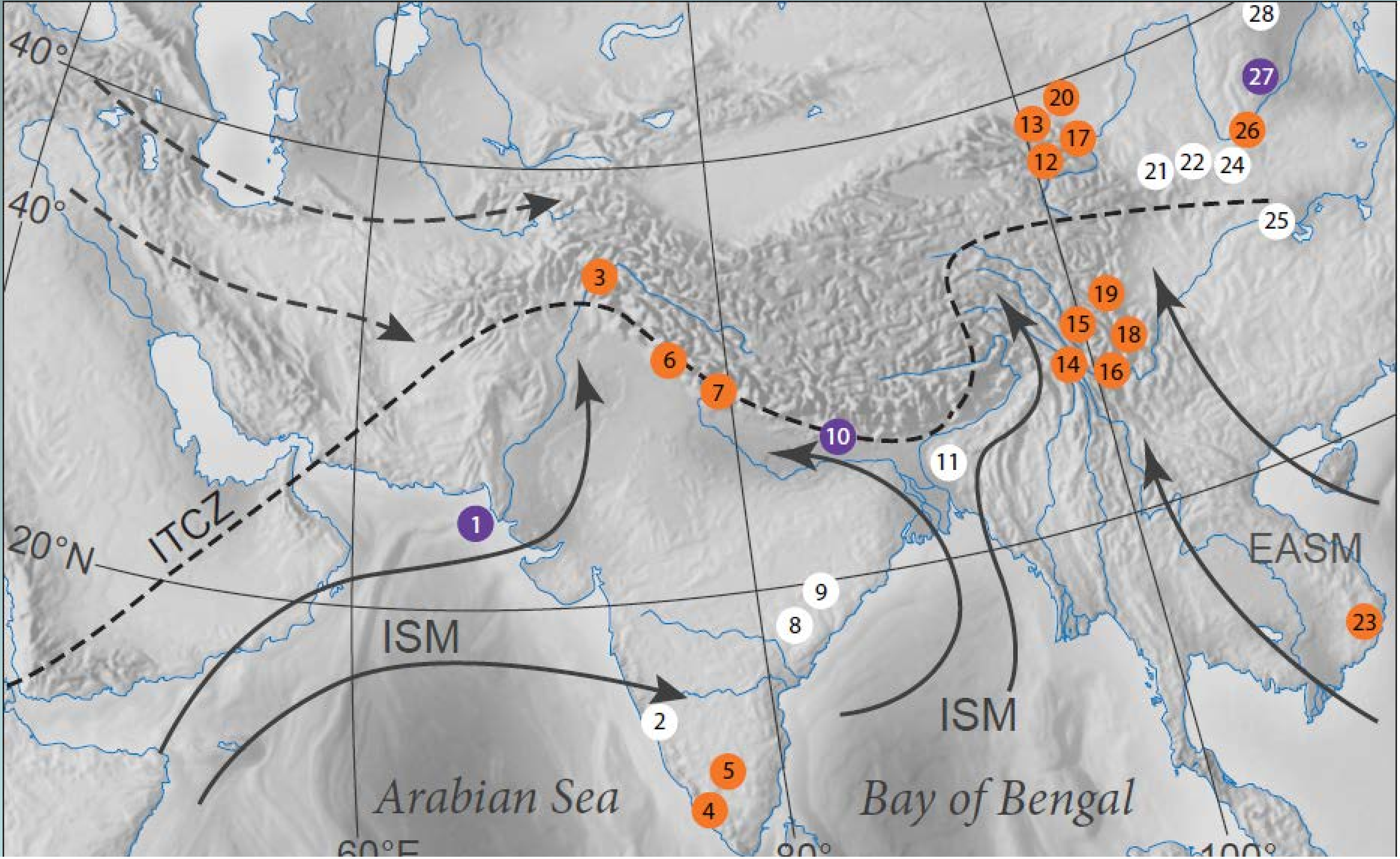
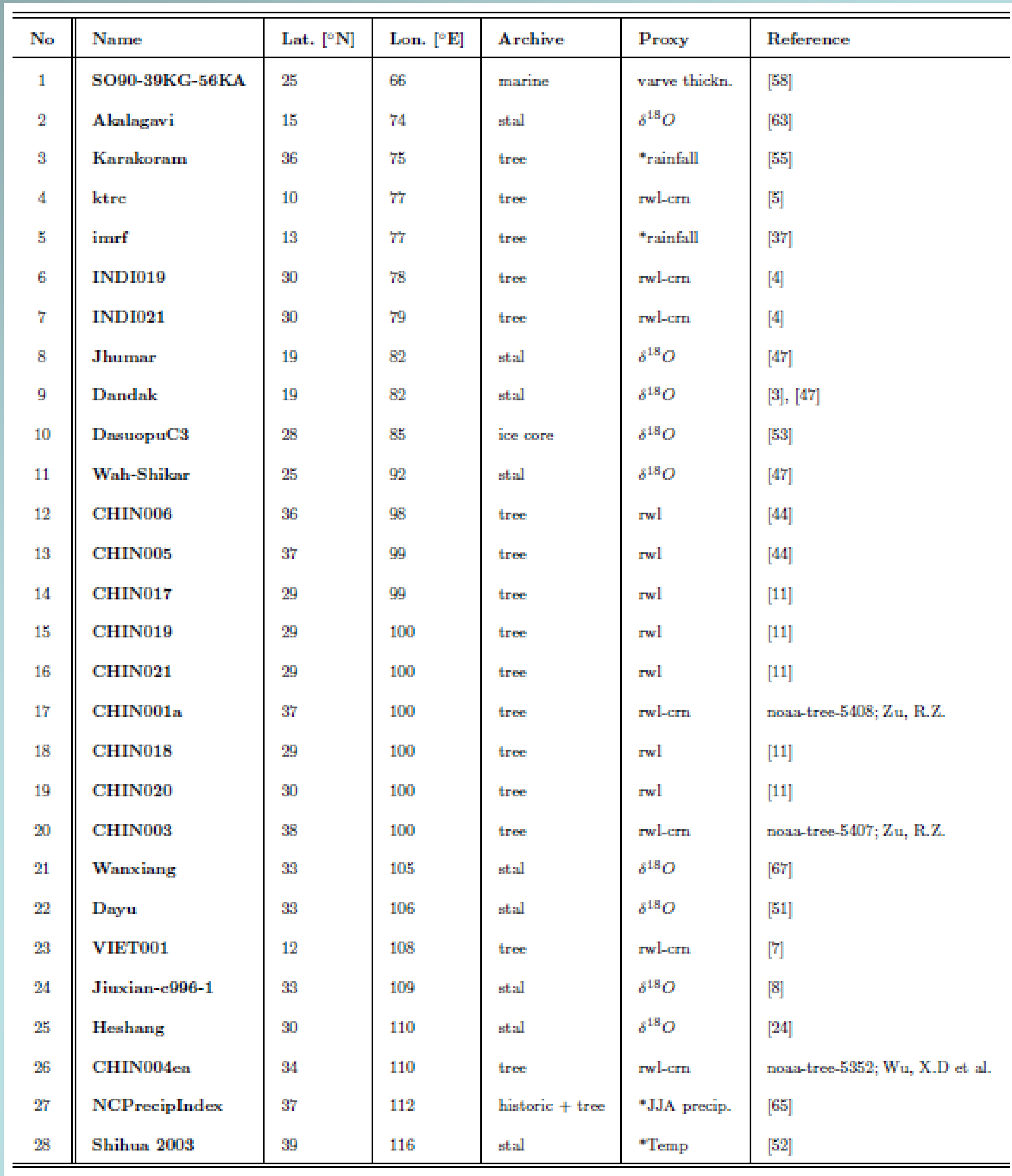


Fig. 1: Study area with generalized summer wind directions of the ISM and EASM (grey arrows), the westerlies (dashed arrows), as well as the spatial coverage of the records considered in the palaeoclimate networks. Numbers of the nodes were assigned according to the longitude of the respective study site and furthermore refer to the entries in Tab. 1. Sites that are at close proximity might show displaced to prevent overlap of the dots and labels. Colors of the dots indicate the type of archive: orange – tree sites, white – stalagmites, purple – other archives (marine sediment (1), ice core (10), reconstruction using historic documents and tree ring data (27)).



No	Name	Lat. [°N]	Lon. [°E]	Archive	Proxy	Reference
1	**SO90-39KG-56KA**	25	66	marine	varve thickn.	[58]
2	**Akalagavi**	15	74	stal	$\delta^{18}O$	[63]
3	**Karakoram**	36	75	tree	*rainfall	[55]
4	**ktrc**	10	77	tree	rwl-crn	[5]
5	**imrf**	13	77	tree	*rainfall	[37]
6	**INDI019**	30	78	tree	rwl-crn	[4]
7	**INDI021**	30	79	tree	rwl-crn	[4]
8	**Jhumar**	19	82	stal	$\delta^{18}O$	[47]
9	**Dandak**	19	82	stal	$\delta^{18}O$	[3], [47]
10	**DasuopuC3**	28	85	ice core	$\delta^{18}O$	[53]
11	**Wah-Shikar**	25	92	stal	$\delta^{18}O$	[47]
12	**CHIN006**	36	98	tree	rwl	[44]
13	**CHIN005**	37	99	tree	rwl	[44]
14	**CHIN017**	29	99	tree	rwl	[11]
15	**CHIN019**	29	100	tree	rwl	[11]
16	**CHIN021**	29	100	tree	rwl	[11]
17	**CHIN001a**	37	100	tree	rwl-crn	noaa-tree-5408; Zu, R.Z.
18	**CHIN018**	29	100	tree	rwl	[11]
19	**CHIN020**	30	100	tree	rwl	[11]
20	**CHIN003**	38	100	tree	rwl-crn	noaa-tree-5407; Zu, R.Z.
21	**Wanxiang**	33	105	stal	$\delta^{18}O$	[67]
22	**Dayu**	33	106	stal	$\delta^{18}O$	[51]
23	**VIET001**	12	108	tree	rwl-crn	[7]
24	**Jiuxian-c996-1**	33	109	stal	$\delta^{18}O$	[8]
25	**Heshang**	30	110	stal	$\delta^{18}O$	[24]
26	**CHIN004ea**	34	110	tree	rwl-crn	noaa-tree-5352; Wu, X.D et al.
27	**NCPrecipIndex**	37	112	historic + tree	*JJA precip.	[65]
28	**Shihua 2003**	39	116	stal	*Temp	[52]

Little Ice Age

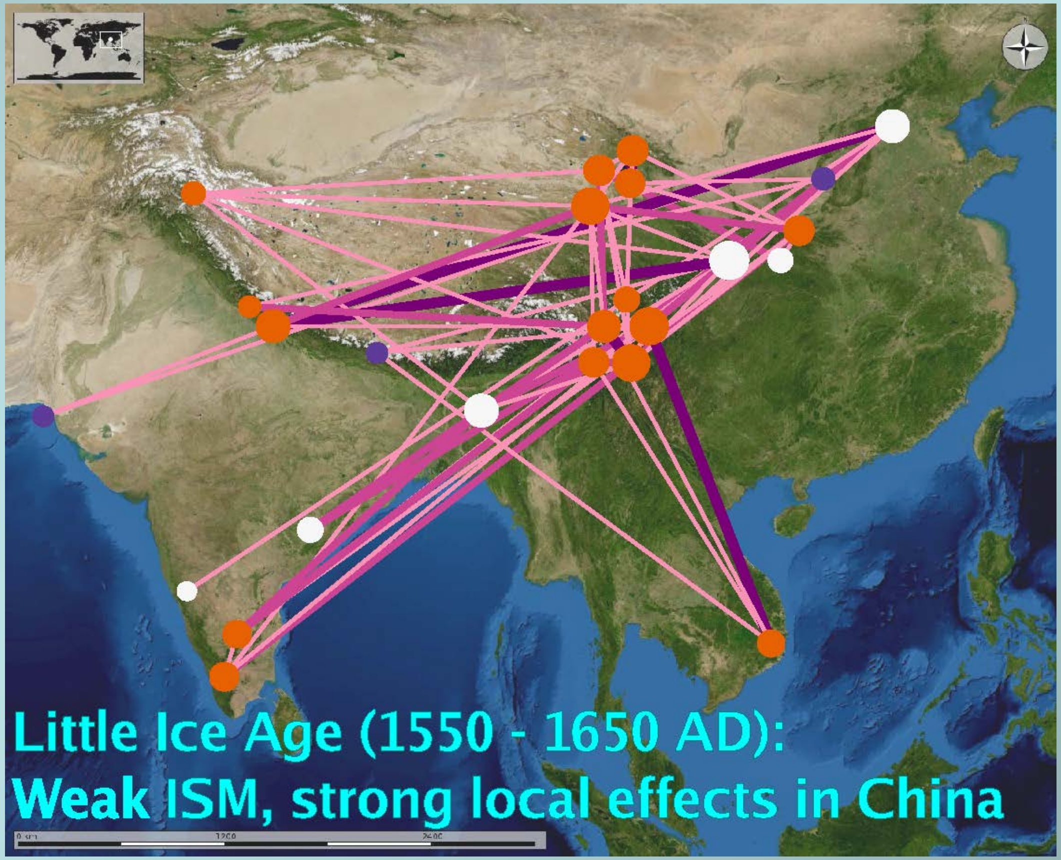
- ISM weaker
- Weaker influence on EASM
- Many short connections (inside China)

Data

White – Stalagmites

Orange – Tree rings

Pink – Ice core



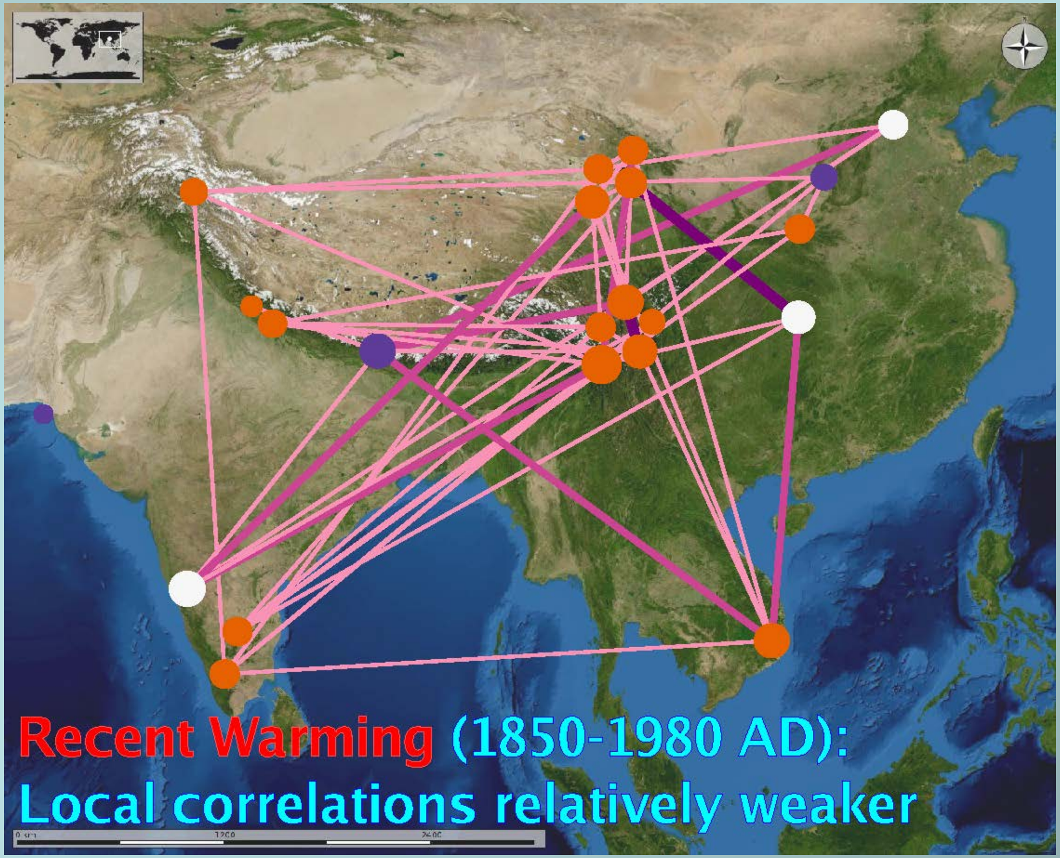
Little Ice Age (1550 - 1650 AD):

Weak ISM, strong local effects in China

Recent Warm Period

- Few short connections

- Strong influence of ISM on EASM



Recent Warming (1850-1980 AD):
Local correlations relatively weaker

Recent Monsoonal Rainfall over South Asia

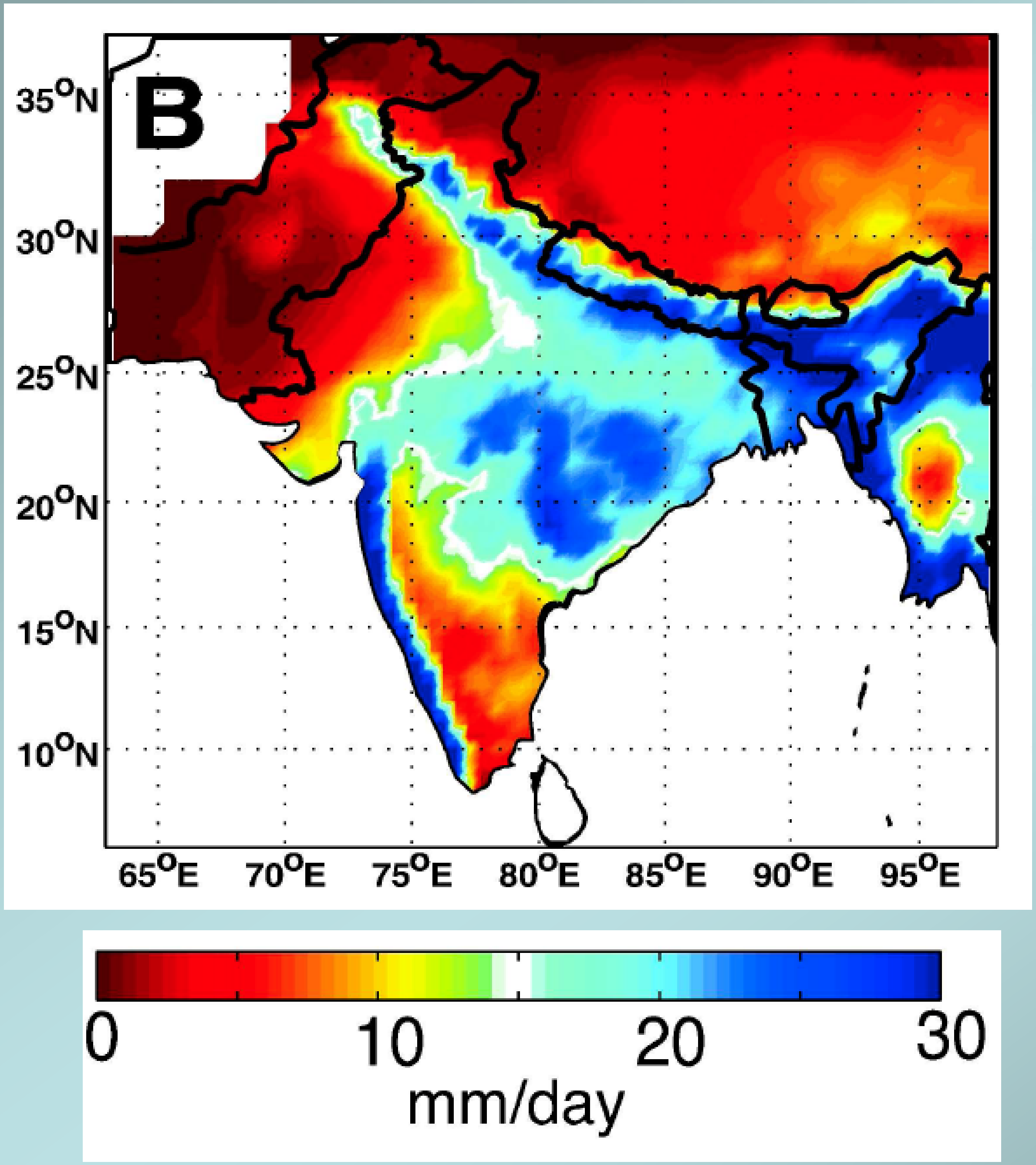
Strongest daily local events

Method: Event Synchronization and Complex Networks

Clim. Dynamics (2011), Europhys. Lett. (2012)

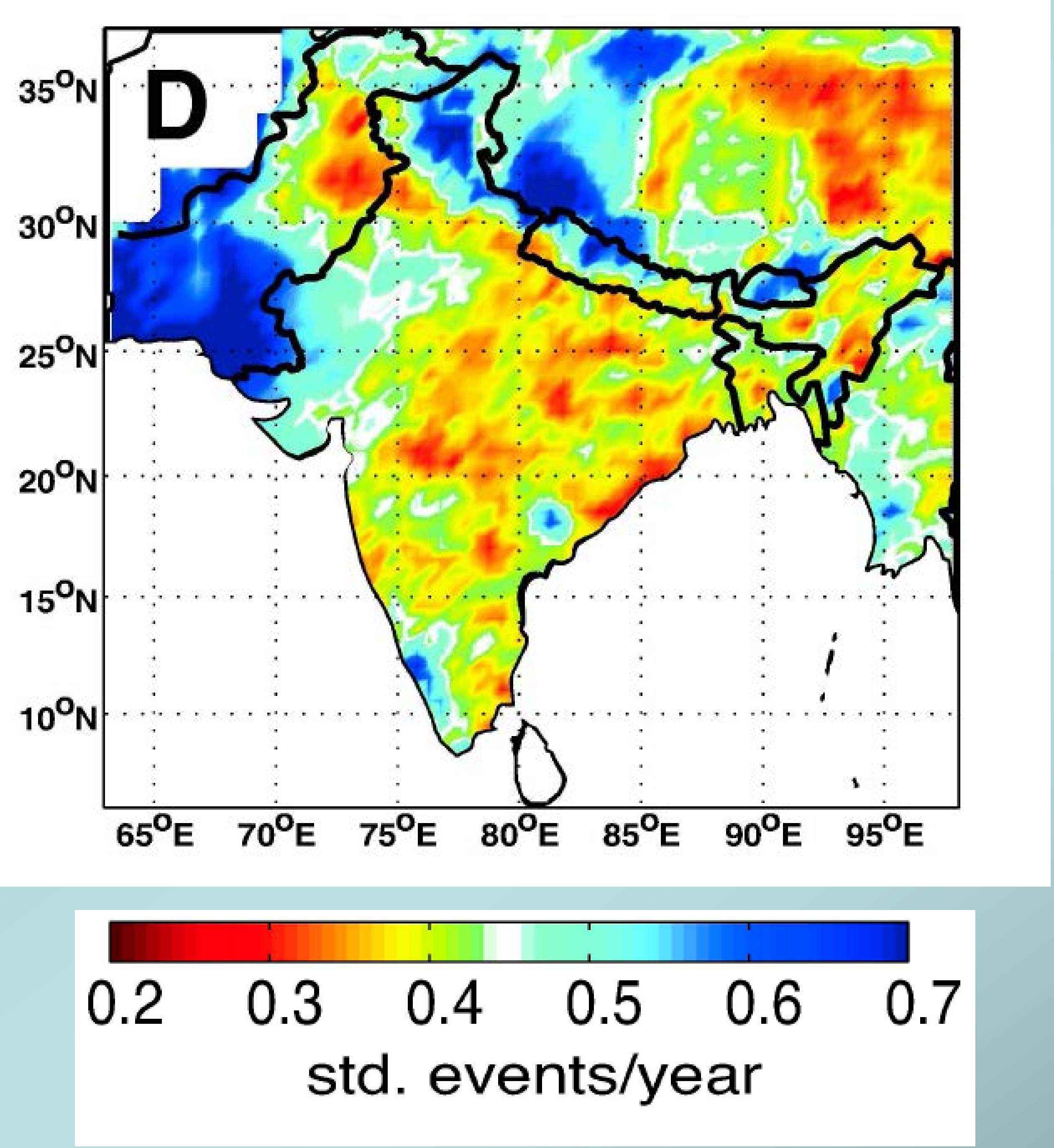
Data

- Asian Rainfall highly resolved observational data integration towards the evaluation of water resources (APHRODITE)
- 1951-2007
- Resolution: 55 km
- Daily data
- Summer period (JJAS – June - September)



Daily rainfall amount for threshold 90% (1951-2007)
- strong events

orographic barriers (Himalaya)



Annual variability

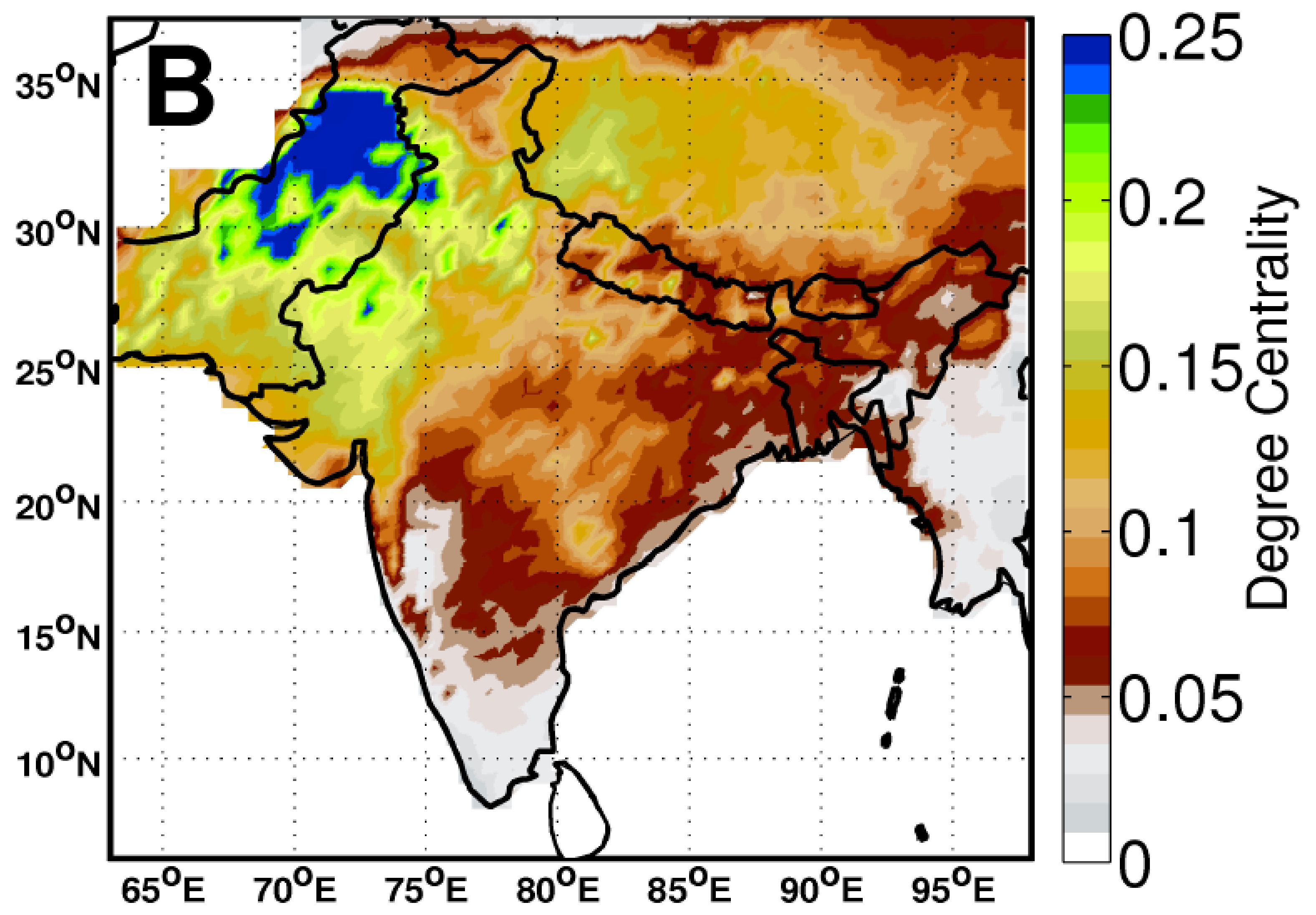
Network construction

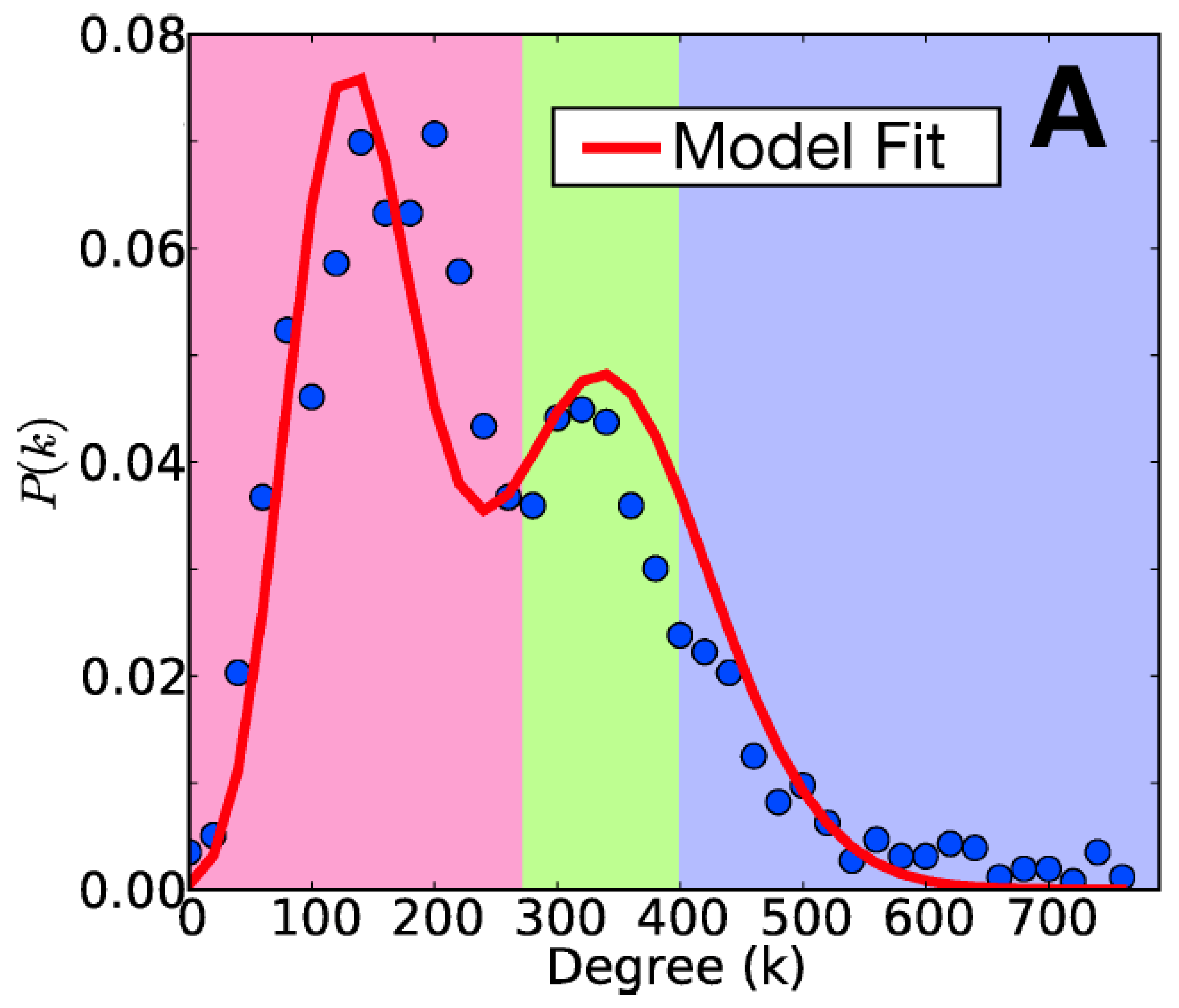
- Event synchronization – m-th strong event occurs at grid positions i and j at time t(m,i) and t(m,j)
- $c(i|j)$ number of times an event occurs at i after it appears in j
- $Q_{ij} = \frac{c(i|j) + c(j|i)}{\sqrt{s_i s_j}}$ strength of event synchronization

Network construction

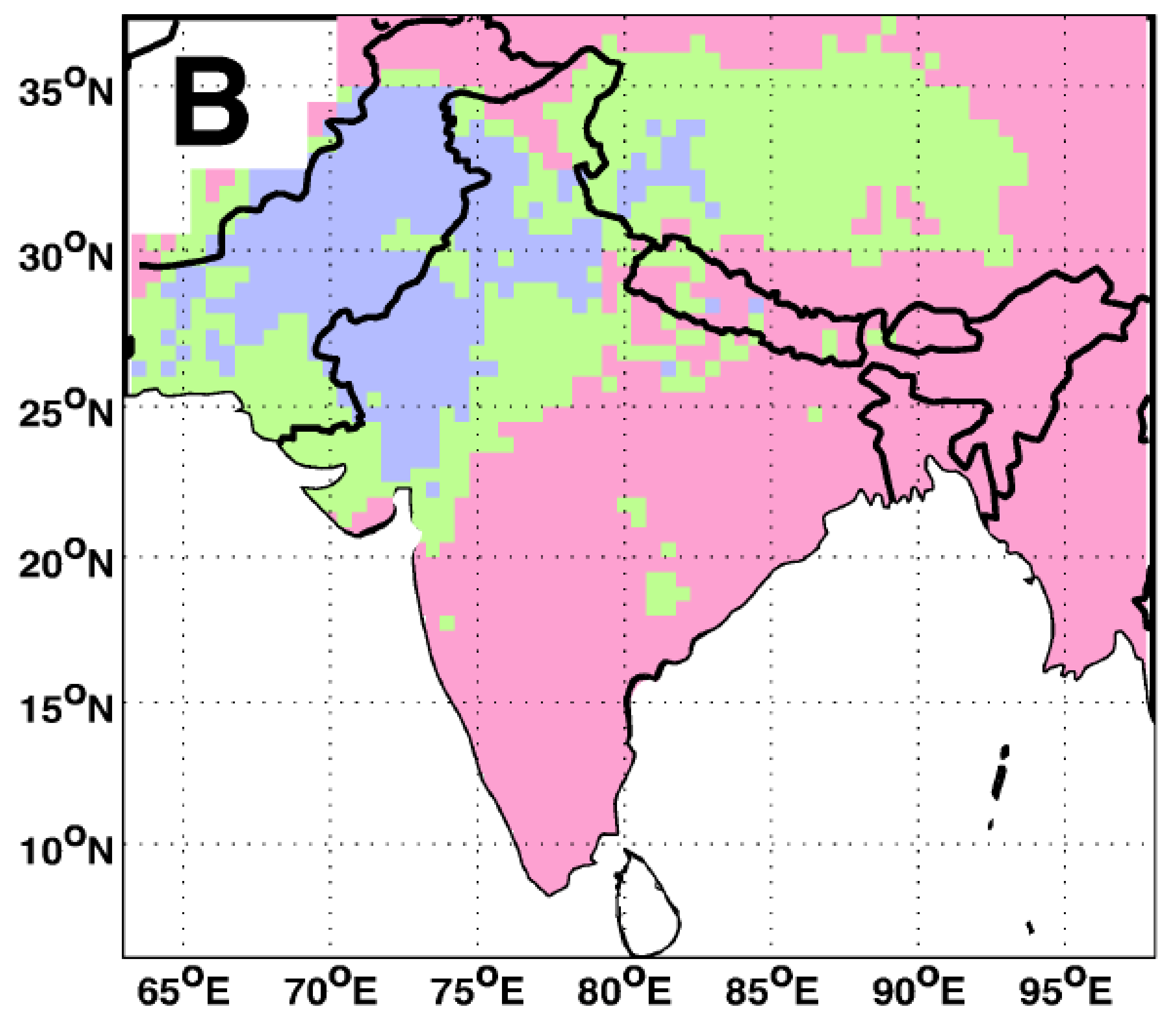
- Adjacency matrix

$$A_{ij} = \begin{cases} 1 \quad if \quad Q_{ij} > \theta_{ij}^{Q} \\ 0 \; else, \end{cases}$$

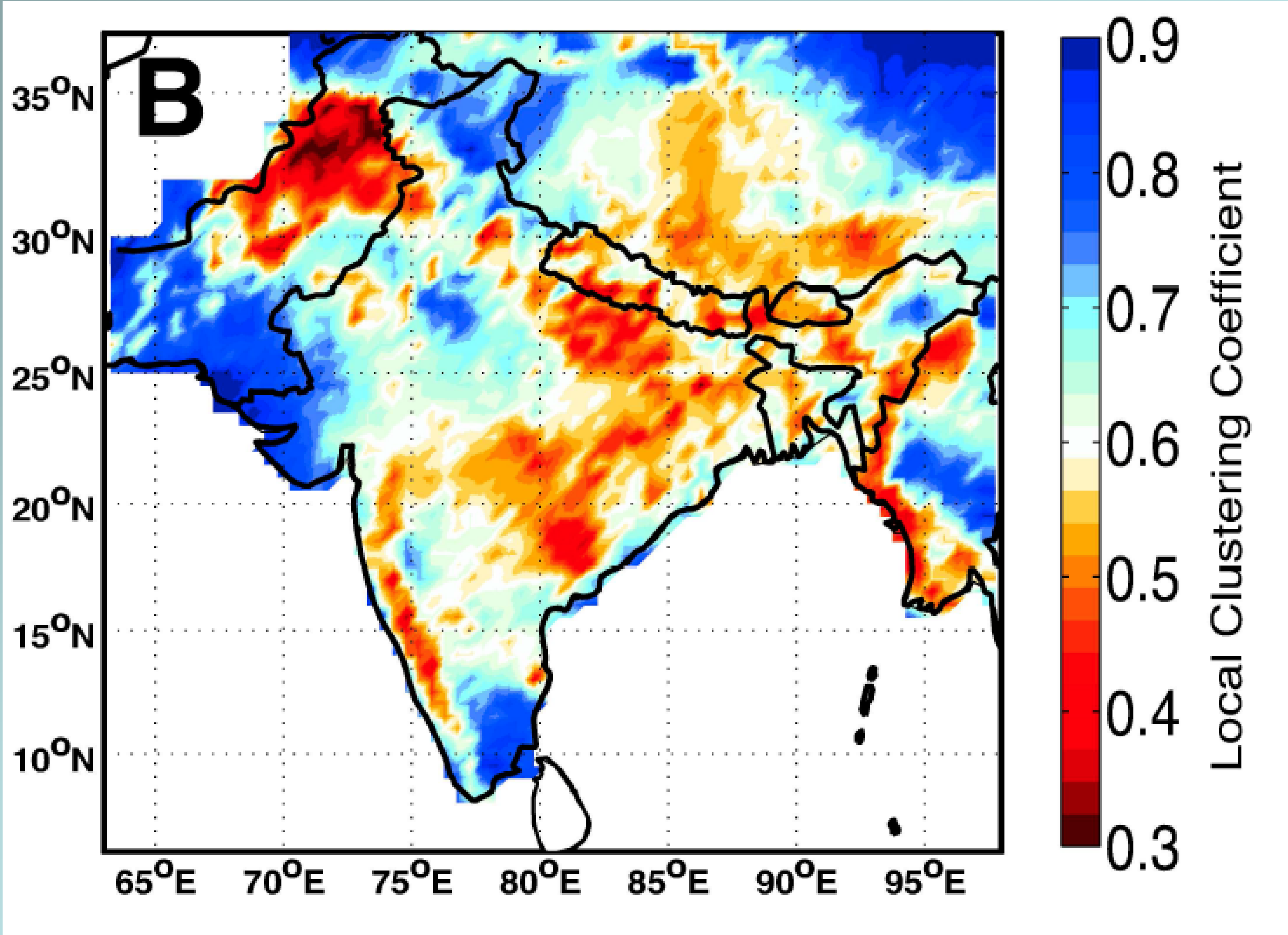

B
35°N
30°N
25°N
20°N
15°N
10°N
65°E
70°E
75°E
80°E
85°E
90°E
95°E
0.25
0.2
0.15
0.1
0.05
0
Degree Centrality



Distribution of degree centrality



Spatial distribution of the different „components“ of P(k)



Local clustering coefficient

➔ Implies **Predictability**

Conclusions

- Complex network approach gives (new) insights into the spatio- (temporal) organization of a complex system as climate
- Helpful vizualization tool but also enables us to identify critical (vulnerable) regions, anomalous behaviour (Monsoon years), to uncover major transport (of moisture), to evaluate predictability of strong events
- Useful for evaluation of forecast techniques?
- Many open problems from methodological and applied viewpoints

Our papers on climate networks

- Europ. Phys. J. Special Topics,174, 157-179 (2009)
- Europhys. Lett. 87, 48007 (2009)
- Phys. Lett. A 373, 4246 (2009)
- Phys. Rev. E 81, 015101R (2010)
- New J. Phys. 12, 033025 (2010)
- Phys. Rev. Lett. 104, 038701 (2010)
- Geoph. Res. Lett. 38, L00F04 (2011)
- PNAS 108, 202422 (2011)
- Europ. Phys. J. B 10797-8 (2011)
- Climate Dynamics 39, 971 (2012)
- Europhys. Lett. 97, 40009 (2012)
- Phys. Rev. Lett. 108, 258701 (2012)
- Climate Dynamics DOI 10.1007/s00382-012 (2012)
- Climate Past (in press, 2012)

Conclusions

- Complex network approach gives (new) insights into the spatio- (temporal) organization of complex systems as climate, brain, power grid,communication…
- Enables us to identify critical (vulnerable) regions, anomalous behaviour (Monsoon years), to evaluate predictability of strong events

Outlook

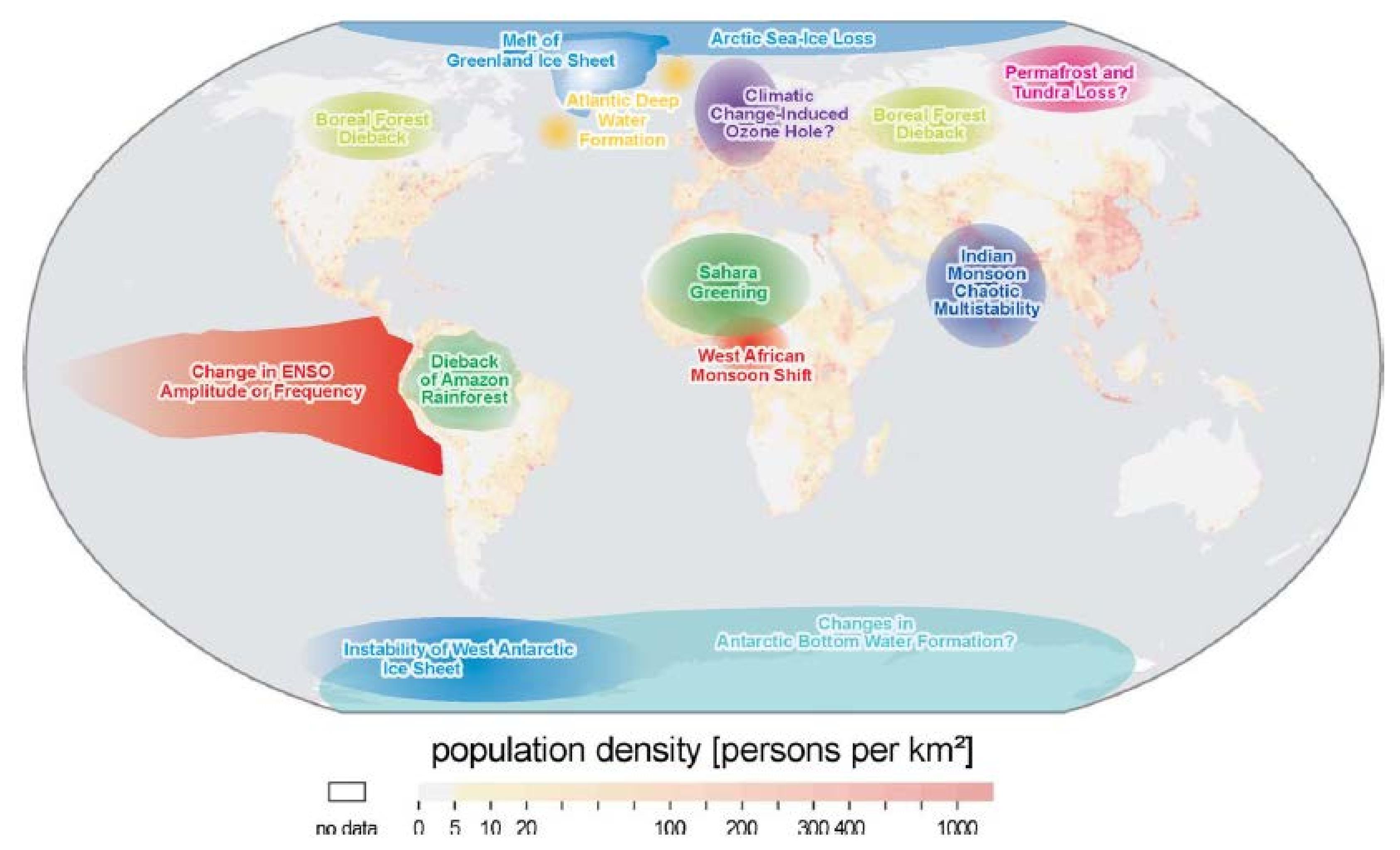
- Network of networks approach promising for describing/modelling important aspects of Earth System´s Dynamics
- However, this is in the beginning and there are basic open questions, e.g.:

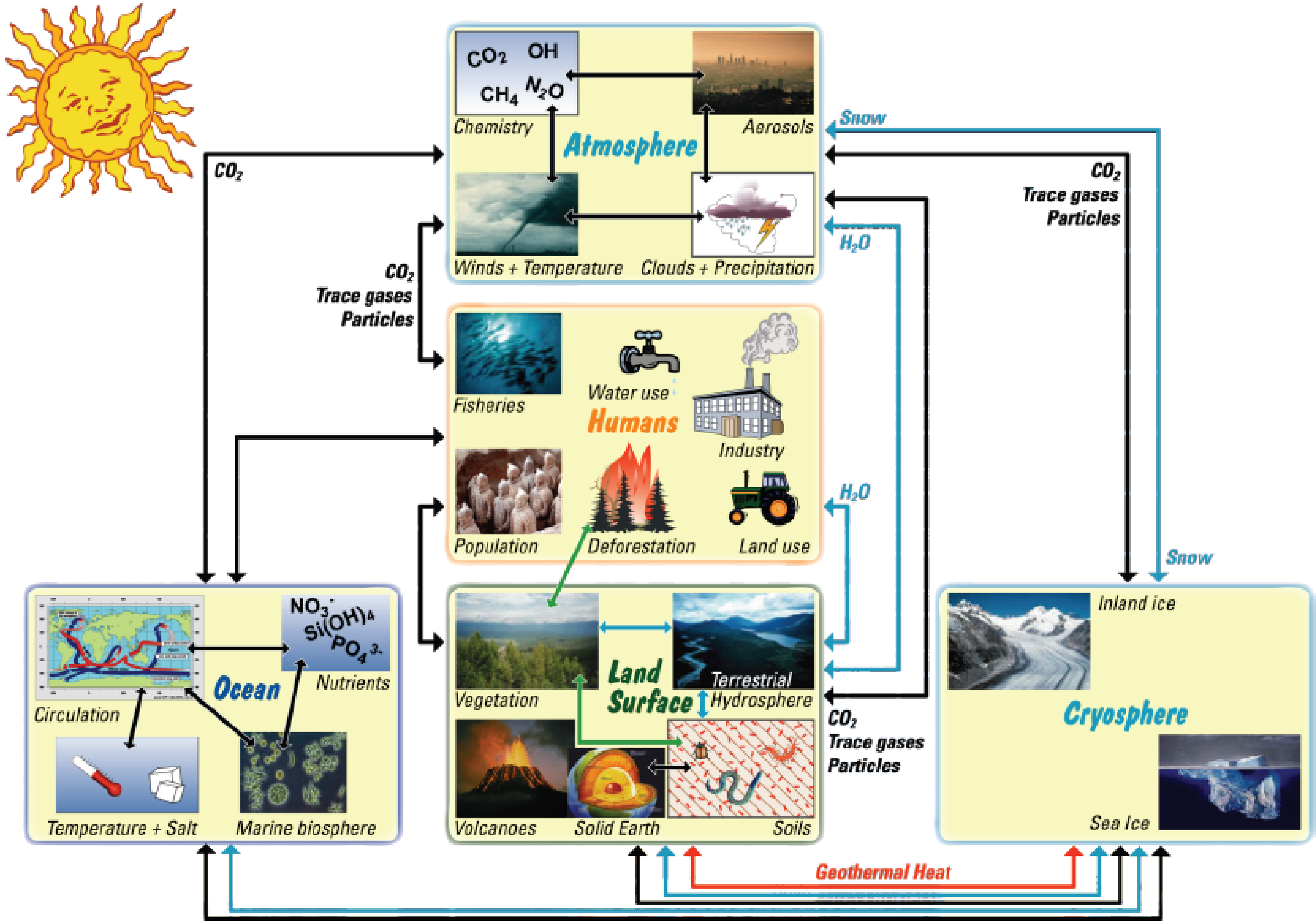
  - How to connect different subsystems (low change in climate – fast changes in society)?

  - Role of critical points (tipping)?

  - Many more: to discuss in working group!

Tipping points (Lenton et al., 2008)




CO₂ OH
CH₄ N₂O
Chemistry
Aerosols
Atmosphere
Winds + Temperature
Clouds + Precipitation
Snow
CO₂
Trace gases
Particles
H₂O
CO₂
CO₂
Trace gases
Particles
Fisheries
Water use
Humans
Industry
Population
Deforestation
Land use
H₂O
Snow
NO₃⁻
Si(OH)₄
PO₄³⁻
Nutrients
Ocean
Circulation
Temperature + Salt
Marine biosphere
Vegetation
Land
Surface
Terrestrial
Hydrosphere
CO₂
Trace gases
Particles
Volcanoes
Solid Earth
Soils
Inland ice
Cryosphere
Sea Ice
Geothermal Heat